# Multihoming and oligopolistic platform competition*

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**Abstract**

We provide a general framework to analyze competition between any number of symmetric two-sided platforms, in which buyers and sellers can multihome, and platforms compete on transaction fees charged on both sides. The framework allows buyers and sellers to have heterogenous benefits from using platforms for transactions, and additionally, buyers to have idiosyncratic preferences over using the different platforms. We show how key primitives such as the number of platforms, the fraction of buyers that find multihoming costly, the value of transactions for buyers and sellers, and the degree of user heterogeneity jointly determine the level and structure of platform fees. Even though having more platforms always reduces the total fee charged to the two sides, whether it shifts the fee structure in favor of buyers or sellers depends on whether most of the buyers are singlehoming or multihoming.

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## 1 Introduction

A growing number of two-sided platforms intermediate transactions between buyers and sellers (or providers) of products and services. Ride-hailing platforms (Uber and Lyft), meal/grocery delivery platforms (Doordash, Grubhub, Postmates and UberEats), hotel booking platforms (Booking.com and Expedia), e-commerce marketplaces (Amazon, Lazada, Shopee, Taobao), and payment card platforms (AMEX, MasterCard and Visa), are well known examples. Our interest in studying these markets stems from the observation that these markets have matured with multiple platforms competing head-to-head, and with no sign of tipping to any one player.

There is by now a large literature on multi-sided platforms, which we will briefly review later. The examples of two-sided platforms listed above have several key features that are under-examined in the existing literature: (i) oligopolistic platforms compete by charging transaction based-fees on each side; (ii) sellers are free to join multiple competing platforms (a phenomenon known in the literature as "multihoming"), which they typically do, and (iii) buyers are free to join multiple competing platforms, and to decide which of these platforms to complete a transaction on, if any. In particular, it has become increasingly easy for users on both sides to multihome, following advancements in tools that make it easier for buyers to compare the options across multiple platforms.[1]

In our framework, which is built upon the seminal contribution by Rochet and Tirole (2003), users (buyers and sellers) have heterogenous valuations over transaction (or interaction) benefits, and platforms charge users on each side per-transaction fees. In the baseline setup, all users can costlessly join multiple platforms. Platforms are differentiated from the buyers' perspective, but are identical from the sellers' perspective. This captures the fact that in many two-sided market settings, sellers view competing platforms as more or less homogenous, while buyers usually have idiosyncratic preferences for using particular platforms over others.

We focus on the equilibrium fees that emerge from platform competition. The preference buyers have towards using certain platforms (i.e., buyer loyalty to each platform) means that even though all buyers and sellers are multihoming, each platform has some market power over sellers. This reflects that sellers may lose too much business if they try to divert buyers to transact through lower fee platforms by delisting from platforms that charge more. If buyers have high platform loyalty, the equilibrium resembles a "competitive bottleneck" type outcome (Armstrong, 2006; Armstrong and Wright, 2007) in which platform competition is focused on attracting buyers and exploiting sellers. Total fees are also high in this case. If, on the other hand, buyers have very little platform loyalty, it is easy for sellers to divert buyers to use the lowest-fee platform to make transactions without worrying about buyers dropping out. In this case platforms have little market power over sellers (so seller fees tend to be low relative to buyer fees) and total fees are competed down close to cost. Thus, our framework highlights that the specific homing patterns of buyers and sellers do not automatically lead to specific

[1] For example, in the ride-hailing market, advancements in mobile phone technology and fare-comparison "metasearch" aggregators such as Google Maps, BellHop and RideGuru, allow more riders to easily compare fares across different ride-hailing apps, resulting in more active multihoming by riders. Similar aggregators have also become quite widely used for hotel booking platforms (Kayak and Trivago) and are currently emerging for food delivery platforms (Foodboss and Mealme).

market outcomes because one also has to take into account users' preferences for transacting on certain platforms and not others. Notably, such a feature is absent in membership-based platform models.

Our first major result is on how increased platform competition (i.e. entry) affects the platforms' equilibrium total fees (the sum of fees charged on both sides) and the fee structure (the allocation of fees across the two sides). We find competition always decreases the total fee. This result reflects two effects, and their interaction with the cross-subsidization mechanism in two-sided markets. First, platform entry intensifies buyer-side competition by making platforms more substitutable for buyers. Second, greater platform substitutability implies a lower buyer loyalty to each platform, which intensifies seller-side competition because sellers can divert more buyers to use low seller-fee platforms when they quit a high seller-fee platform. Clearly, both effects decrease the total fee. As for the fee structure, we find that, under fairly general conditions, seller-side competition tends to dominate and so the seller fee decreases. The lower seller fee then implies a smaller profit margin on the seller side from attracting buyers (i.e., a weaker incentive to cross-subsidize buyers), so that platforms increase their buyer fees in response.

Our second set of results explore what happens if some buyers face a cost to multihome. This gives rise to a partial multihoming equilibrium with a mix of multihoming and singlehoming users on the buyer side. Starting from our baseline setting with two-sided multihoming, a decrease in the fraction of buyers multihoming decreases fees to buyers, and increases fees to sellers and total fees. Intuitively, having fewer multihoming buyers makes it harder for sellers to divert buyers' transactions, increasing the platforms' market power over sellers. As a corollary, this result implies that buyer multihoming reduces the tendency for high seller fees that typically arises in the competitive bottleneck case when buyers all singlehome. Furthermore, our result is obtained in a setting in which user heterogeneity is with respect to transaction benefits and fees are per transaction, rather than these being membership based as in the previous literature looking at the implications of multihoming, e.g., Belleflamme and Peitz (2019a).

Interestingly, we find there is a non-trivial interaction between buyers' homing behavior and the effects of platform competition. Even though increased platform competition always reduces the total fee charged to the two sides, whether it shifts the fee structure in favor of buyers or sellers depends on whether most of the buyers are singlehoming or multihoming. When most of the buyers multihome, increased platform competition induces platforms to compete more intensely for sellers, as in the baseline setup. However, when most of the buyers singlehome, platforms have monopoly power over providing access to their buyers for the multihoming sellers and so entry induces platforms to compete more intensely for buyers rather than for sellers.

Our third set of result explores various determinants of buyer loyalty to platforms, which, as noted above, play an important role in our equilibrium characterization. We focus on changes in the distribution of buyer and seller preferences. First, increasing the value buyers put on transacting with sellers (relative to not transacting) means that buyers will care more about whether a transaction occurs and less about the choice of which platform carries out the transaction. This indicates a lower buyer loyalty to platforms, which lowers the platforms' market power over sellers. A similar reasoning applies if we increase buyer heterogeneity when the buyer-side fee is positive. A greater heterogeneity dampens buyers' sensitivity towards the net cost of using platforms, meaning they become more willing to transact through any of the platforms, which indicates a lower loyalty and a lower market power over sellers. Meanwhile, increasing the value sellers put on transacting or seller heterogeneity has the opposite effect, giving platforms more market power over sellers.

### Relevant literature

The literature on two-sided markets starts with the seminal papers by Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), and Armstrong (2006), which provide a basic foundation for studying pricing schemes by monopoly and duopoly platforms.[2] Among these papers, our study is closest to Rochet and Tirole (2003), which we build on by allowing for more than two platforms to compete, transaction-specific buyer surpluses, and richer homing behavior on the buyer side. A key modelling difference is we explicitly model the underlying distribution of buyers' and sellers' valuations over interaction benefits rather than expressing the formula in terms of reduced-form demand functions. Our micro-founded approach offers three key benefits. First, it clarifies the nature of buyer loyalty to each platform in terms of buyer multihoming costs, platform differentiation, and user interaction values. Second, it provides comparative static results with respect to platform entry for any number of competing platforms. And third, it allows us to draw comparisons with the "competitive bottleneck" literature, revealing a non-trivial interaction between the effects of platform entry and homing behaviors of buyers.[3]

In developing and investigating a model of oligopolistic platform competition, our study relates closely to the recent contribution by Tan and Zhou (2021) and Anderson and Peitz (2020). Tan and Zhou (2021) presents a model of oligopolistic multi-sided platform competition rooted in the membership pricing model of Armstrong (2006). They provide important insights on the impact of platform entry and on the extent of excessive or insufficient platform entry. However, their framework focuses on singlehoming users on both sides and does consider transaction fees and heterogeneity in interaction benefits, which are the focus of our study. Anderson and Peitz (2020) analyze oligopolistic media markets with singlehoming viewers and multihoming advertisers (analogous to buyers and sellers in our setup, assuming ad-loving viewers). In their two-sided pricing extension, the platform sets a per-viewer fee on the advertiser side and a participation fee on the viewer side. They show that both sides are better off with platform entry, consistent with our observations in Section 4.2 even though we allow both sides of users to multihome. Nonetheless, the underlying economic reasoning behind their result is quite different because viewers in their model make only participation decisions while buyers in our model make transaction decisions in addition. [4]

[2]Subsequent developments in the two-sided market literature extend the canonical two-sided framework in various directions. Among others, Weyl (2010) provides a more general model of a monopoly two-sided platform and examines the source of welfare distortions in platform pricing; Hagiu (2006) considers platform pricing and commitment issues when two sides of the market do not participate simultaneously; Jullien and Pavan (2019) consider platform pricing under dispersed information; Karle et al. (2020) explore how the phenomenon of platform market tipping relates to the presence of seller competition on platforms.

[3]In the subsequent sections, we also compare our results in Propositions 1 and 3 to those obtained by Rochet and Tirole.

[4]In a slightly different vein, Correia-da Silva et al. (2019) and Adachi et al. (2022) consider homogeneous

As mentioned above, when buyers are loyal to particular platforms, our results resemble the classic "competitive bottleneck" result obtained by Armstrong (2006), Armstrong and Wright (2007), and recently revisited by Belleflamme and Peitz (2019a). These studies typically start with a configuration of singlehoming on both sides, and show that multihoming on one side leads to a competitive bottleneck, whereby platforms no longer need to compete for the multihoming side due to the monopoly power over providing exclusive access to each (singlehoming) user on the other side. Thus, in these studies, buyer-side multihoming would shift the fee structure in favor of sellers by shutting down competition on the buyer side. In contrast, we specify that buyers and sellers are always free to multihome in our baseline setting, and explore factors that affect buyer loyalty to platforms (including whether some buyers only singlehome or some buyers have strong preferences to only transact on particular platforms).

One paper that does consider two-sided multihoming is Bakos and Halaburda (2020), although also in Armstrong's framework. They compare two-sided multihoming with the benchmarks of two-sided singlehoming and competitive bottleneck, showing that two-sided multihoming eliminates the strategic interdependence between the two sides in platform pricing (so that the two platforms have no incentive to cross-subsidize across the two sides). A key ingredient for their result is that the market is fully covered on both sides. In our framework, in which the market is not fully covered on either side, strategic interdependence is restored. Jeitschko and Tremblay (2020) consider a model with heterogenous interaction benefits but assume that platforms charge membership fees. They show that a variety of possible homing configurations can arise in the equilibrium, including the case with a mix of multihoming and singlehoming on both sides of the market. Such an equilibrium multiplicity does not arise in our setup because we focus on platforms that charges transaction fees, which constitute an "insulating tariff" (Weyl, 2010) whereby users' participation decisions are independent of the (expected) mass of participating user on the opposite site.

At a more general level, our analysis on the impact of user multihoming behavior and its interaction with platform entry relates to several recent papers in the media literature that investigate similar issues (Ambrus et al., 2016; Athey et al., 2018; Anderson et al., 2019). Ambrus et al. and Athey et al. show that multihoming by media consumers can either increase or decrease the equilibrium number of ads that platforms admit, depending on the correlation of consumers preference and the extent to which advertisements generate negative externalities on consumers. Anderson et al. consider a model of multihoming media consumption based on the Salop (1979) circular city model, deriving the interesting property of "incremental value pricing" whereby platform entry has no effect on consumers but harms advertisers.

The rest of the paper proceeds as follows. Section 2 lays out the main model, the equilibrium of which is characterized in Section 3. Section 4 investigates the impact of platform entry, Section 5 investigates buyer multihoming cost and the interaction with platform entry, while Section 6 investigates various other factors that influence buyer loyalty. Finally, Section 7 concludes. All proofs and omitted derivations are relegated to the Appendix.

oligopolistic two-sided platforms that compete in a Cournot setup in which the platforms commit to quantity choices and the membership prices on each side adjust to clear the market.

## 2 Model setup

There is a set $\mathbf{N} = \{1, .., n\}$ of $n \geq 1$ platforms which compete for a continuum of buyers and a continuum of sellers, both of measure one. Buyers and sellers wish to "interact" or "transact" with each other to create economic value. If we consider any buyer/seller pair, then we can assume without loss of generality that each such pair corresponds to one potential transaction. Each transaction must occur through one of the platforms, and it can occur only if there exists at least one platform that both sides of the buyer-seller pair join.[5] Let $\mathbf{p}_i = \left(p_i^b, p_i^s\right)$ denote the fees charged by platform $i$ to buyers and sellers for each transaction facilitated.

□ **Sellers.** Following Rochet and Tirole (2003), we assume that seller surpluses do not vary across platforms. Each seller is indexed by a draw of per-transaction surplus, $v$. The net seller utility from each transaction through platform $i$ is $v - p_i^s$, while the utility from not transacting is normalized to zero. Specifically, $v \in [\underline{v}, \bar{v}]$ (where $\underline{v} \geq -\infty$ and $\bar{v} \leq \infty$) is drawn i.i.d across sellers from cumulative distribution function (CDF) $G$ with density function $g$, in which $1 - G$ is log-concave.

□ **Buyers.** Transaction decisions are endogenously initiated by buyers. For each potential transaction with a given seller $v$, if a buyer transacts with the seller and does so through platform $i \in \mathbf{N}$, the net utility is

$$b_0 + \epsilon_i - p_i^b.$$

Here, $b_0$ is a buyer-seller specific component that measures the intensity to which a buyer desires a transaction (gross value for transaction) with the seller. It is drawn i.i.d across buyers and transactions, but invariant across platforms. Then, $\epsilon_i$ is a buyer-platform match component that measures buyer preference for transaction platforms, and it is drawn i.i.d across buyers and platforms. Utility provided by the outside option (of not transacting) is zero. For all $i \in \mathbf{N}$, let $F$ be the common CDF for $\epsilon_i \in [\underline{\epsilon}, \bar{\epsilon}]$ (where $\underline{\epsilon} \geq -\infty$ and $\bar{\epsilon} \leq \infty$) with log-concave density $f$. To simplify the notation and avoid the need to carry a negative sign throughout our analysis, denote $\epsilon_0 \equiv -b_0$ and $F_0$ be the CDF for $\epsilon_0 \in [\underline{\epsilon}_0, \bar{\epsilon}_0]$ with log-concave density $f_0$. Finally, we assume that $v$, $\epsilon_0$, and $\epsilon_i$ for $i \in \mathbf{N}$ are independently drawn.

□ **Platforms.** We allow transaction fees, $\mathbf{p}_i = \left(p_i^b, p_i^s\right)$, to be negative (e.g negative buyer fees in the case of rewards in payment platforms, and negative seller fees in the case of ride-hailing apps' payments to riders). Facilitating each transaction involves a marginal cost of $c$, which is assumed to be constant and symmetric across all platforms. We focus on the transactional aspect of platforms and abstract from any participation benefits (or costs) and fees. Therefore, platform $i$'s profit is written as

$$\Pi_i\left(\mathbf{p}_i; \mathbf{p}_{-i}\right) = \left(p_i^b + p_i^s - c\right) Q_i\left(\mathbf{p}_i; \mathbf{p}_{-i}\right),$$

where $\mathbf{p}_{-i}$ is the fees set by all other platforms excluding $i$ while $Q_i$ is the total volume of transactions facilitated by platform $i$, which will be determined in Section 3.

[5] By reframing each transaction as "platform-intermediated transactions" and the outside option below as "direct transactions", the model easily allows for scenarios where buyers and sellers can interact directly, e.g., payment card platforms and cash-based direct transactions. See Rochet and Tirole (2002, 2003) and Bedre-Defolie and Calvano (2013) for such interpretations.

☐ **Participation multihoming.** Both buyers and sellers are always allowed to join multiple platforms. In the baseline, we assume that participation is costless. Note if both sides multihome, the choice of which of these platform to use for a transaction is a-priori indeterminate. Following Rochet and Tirole (2003) and consistent with each of our motivating examples (e.g., ride-hailing services and payment card platforms), we assume that whenever a seller is available on multiple platforms, the buyer is the one that chooses which platform to complete the transaction on.[6]

☐ **Timing and equilibrium.** The timing of the game is summarized as follows:

1. All $n$ platforms simultaneously set their transaction fees with platform $i$'s being $\mathbf{p}_i = \left(p_i^b, p_i^s\right)$;
2. Given the platform fee profile $\mathbf{P} = (\mathbf{p}_1, ..., \mathbf{p}_n)$, sellers and buyers observe all fees and their realized draws $v$ and $(\epsilon_1, ..., \epsilon_n)$ and simultaneously decide which platform(s) to join.[7]
3. For each potential transaction, buyers observe their realized $b_0$ and choose whether to transact, and if so, through which platform.

Note we are assuming buyers' preferences across platforms are fixed but whether they want to make a transaction or not varies with each potential transaction. Our equilibrium concept is pure-strategy subgame perfect Nash equilibrium (SPNE), and we focus only on symmetric equilibria where all platforms set the same fees. As a tie-breaking rule, we assume that, whenever a user is indifferent between joining and not joining a platform, she breaks the tie in favor of joining.

### Discussion of the modelling features

☐ **Model specification.** Certain asymmetries across the two sides are key to our framework: namely, that there are platform-specific shocks across buyers but not across sellers, and buyers rather than sellers choose which platform to use for a transaction. On the other hand, some other asymmetries are not critical. We could easily add an ex-post transaction specific match value $v_0$ for sellers to parallel $b_0$ on the buyer side. That would be redundant given sellers only make joining decisions, and so we drop this for the sake of notational simplicity. Similarly, we could easily add an ex-ante buyer-specific match value $b$ to parallel $v$ on the seller side. That would be redundant as well given we already have ex-post buyer-transaction specific match values $b_0$, and so we also drop this for the sake of notational simplicity. Thus, allowing for ex-ante and ex-post platform-independent shocks for both buyers and sellers is easily incorporated. Our choice that buyers' platform-specific match values are drawn ex-ante rather than (or in addition to) being drawn ex-post does not matter for the baseline model given all buyers multihome, but matters for the extension to partial multihoming. If the shocks are drawn ex-post *rather than* ex-ante, buyers that face a cost to multihome would view platforms

[6] Indeed, the U.S. Supreme Court decision on Ohio v. American Express allows American Express to continue to prevent sellers from steering buyers to rival (cheaper) card platforms using monetary or non-monetary incentives, thereby ensuring it is buyers alone that decide which payment card they will use.

[7] Assuming that buyers do not observe the seller-side fees, which may be more realistic in some cases, does not change our analysis. See also the discussion in Section 5.

as homogenous when choosing which platform(s) to join, which in the partial multihoming case would raise the possibility of a mixed strategy pricing equilibrium. Alternatively, if the shocks are drawn ex-post *in addition to* the existing ex-ante shock, the partial multihoming case is no longer as tractable given the quasi-demand of singlehoming and multihoming buyers would take different forms.

☐ **Illustrative example.** As an illustration, we consider ride-hailing services.[8] Drivers (sellers) are indexed by their type $v$, which is proportional to the differences in utility of the driver between driving and idling. It is typically negative due to the effort and cost involved with driving (recall that $v$ can be negative). The market has a large number of routes, and at any point in time each route is occupied by a driver. Consider a rider (buyer) who wants to travel on a given route that happens to be occupied by a driver $v$. If the driver on this route is unavailable on any of the ride-hailing platforms, the rider is unable to reach this driver and has to use alternative forms of transport (e.g., public transport) to travel on this route, obtaining utility normalized to zero. If the driver is available on platform $i$, then the rider chooses between engaging with the driver or using alternative forms of transport, depending on whether $b_0 + \epsilon_i - p_i^b$ is greater than zero. Here, $b_0$ is the rider-specific value for the convenience of using a ride-hailing service, and $\epsilon_i$ represents idiosyncratic preference for ride-hailing platform $i$. This idiosyncratic preference could reflect brand-affiliation, convenience of using the rider's preferred form of payment on the platform, safety features, the user interface, among other factors. Multihoming riders compare the net utility of the ride across platforms (that have access to the driver), and then order through one of the platforms.

☐ **Network effects.** In our model, there is a network effect at the market level because each buyer (seller) benefits from having access to more sellers (buyers) because there are more transactions that can potentially be made. However, as will be seen below, when sellers multi-home in the equilibrium, this partially shuts down the network effect at the level of individual platforms, in that buyers' ability to access sellers does not depend on sellers being available on a particular platform. However, buyers still benefit from having more sellers available on any given platform, since this increases the likelihood they will be able to make transactions on their preferred platform. Meanwhile, network effects also still play an important role in sellers' participation decisions. Due to the heterogeneity in buyer preferences across platforms, sellers' participation decisions have to take into account the number of unique buyers they can gain access to on each platform $i$ (since otherwise these buyers may not want to buy via other platforms $j \neq i$).

[8] A similar illustration applies to meal/grocery delivery platforms, hotel booking platforms, e-commerce marketplaces, and payment platforms: buyers choose the channel of transaction in all these examples. One complication is that the issue of seller pricing and pass-through arises in these examples. Nonetheless, our model is still applicable so long as (i) sellers do not price discriminate across buyers that transact through different platforms; and (ii) there is a sufficiently large additional group of buyers who always avoid platform-intermediated transactions (see the reinterpretation in footnote 5). See Appendix B of Guthrie and Wright (2003) for a formal proof of this in the context of payment card platforms. Feature (i), also known as "price coherence", is often enforced by platforms, either explicitly through price parity clauses (Edelman and Wright, 2015) or implicitly through ranking algorithms (Hunold et al., 2020).

## 3 Equilibrium analysis

### 3.1 Decisions of buyers and sellers

□ **Choice of transaction medium.** Consider a buyer that has joined a set $\Theta^b$ of platforms who wishes to transact with a seller $v$ that has joined a set $\Theta^v$ of platforms. The buyer can either perform the transaction through one of the of platforms (that the pair has joined in common) or opt for the outside option (not transacting). Thus, the buyer uses platform $i \in \Theta^b \cap \Theta^v$ if

$$b_0 + \epsilon_i - p_i^b > \max_{j \in \Theta^b \cap \Theta^v} \left\{ b_0 + \epsilon_j - p_j^b, 0 \right\}.$$

Given that $\epsilon_0 = -b_0$, the condition becomes

$$\epsilon_i - p_i^b \geq \max_{j \in \Theta^b \cap \Theta^v} \left\{ \epsilon_j - p_j^b, \epsilon_0 \right\}.$$

As will be seen later, we will focus on the participation equilibrium in which all buyers multihome on all platforms, i.e. $\Theta^b = \mathbf{N}$ for all buyers, so that $\Theta^b \cap \Theta^v = \Theta^v$. For each seller $v$, denote the mass of buyers who wish to transact with the seller and do so using platform $i \in \Theta^v$, as

$$B_i^{(\Theta^v)} \equiv \Pr \left( \epsilon_i - p_i^b \geq \max_{j \in \Theta^v} \left\{ \epsilon_j - p_j^b, \epsilon_0 \right\} \right) \text{ for any } \Theta^v \subseteq \mathbf{N}. \quad (1)$$

From (1), note that a seller, by selecting the platform(s) she wants to join, can restrict the set of platforms that buyers can choose from to make their transactions. It should be emphasized that (1), also known as "buyer quasi-demand", does not necessarily equal to the mass of buyers who have joined $i$ given that the transaction choice is endogenous.

Denote the symmetric fee equilibrium under multihoming buyers as $\hat{\mathbf{p}} = (\hat{p}^b, \hat{p}^s)$. We consider a platform $i$ which deviates from the equilibrium and sets $\mathbf{p}_i = (p_i^b, p_i^s) \neq \hat{\mathbf{p}}$. Whenever convenient, we use $\mathbf{N}_{-i} \equiv \mathbf{N} \setminus \{i\}$ to denote the set of all platforms excluding $i$.

□ **Buyer participation.** It is straightforward to see that the assumptions of (i) buyers get to choose the final medium for transaction and (ii) zero joining cost, together, imply that it is a weakly dominant strategy for any given buyer to join all platforms, regardless of the fees set by the platforms.[9] Thus, given our tie-breaking rule, we focus on the participation equilibrium with all buyers multihome on all platforms.

□ **Seller participation.** The profile of seller participation generally depends on how the seller fee set by platform $i$ compares to other platforms. To derive the equilibrium fees, let us focus on the participation profile after an upward deviation by platform $i$, that is, $p_i^s \geq \hat{p}^s$.

Given $p_i^s \geq \hat{p}^s$ and that all platforms $j \neq i$ set the lowest seller fee $\hat{p}^s$, it is clear that a seller either joins no platforms, joins all platforms except $i$ (i.e., $N_{-i}$), or joins all platforms including

[9] More formally, consider a buyer who contemplates joining an additional platform $i$ after already joining another platform $j$. The additional participation on platform $i$ is weakly beneficial for two reasons. First, the buyer gains access to any sellers who are available on platform $i$ but not available on platform $j$, and this additional access is strictly beneficial if $\epsilon_i - p_i^b > \epsilon_0$. Second, even if joining platform $i$ does not provide any additional access, a buyer can switch his transaction over to platform $i$ if it provides a higher utility than transacting through platform $j$, i.e. if $\epsilon_i - p_i^b \geq \epsilon_j - p_j^b$. We relax the assumption of zero joining cost in Section 5.

$i$ (i.e., $N$). For a seller $v$, the net surplus from joining all platforms including $i$ is

$$(v - \hat{p}^s) \sum_{j \in \mathbf{N}_{-i}} B_j^{(\mathbf{N})} + (v - p_i^s)\, B_i^{(\mathbf{N})}$$

as illustrated in Figure 1 below (with $i = 1$ and $\mathbf{N}_{-i} = \{2, 3\}$).

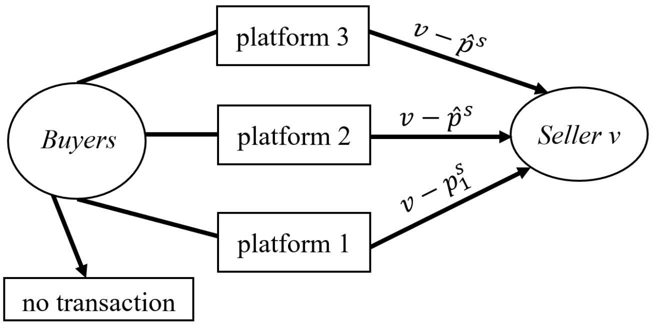


Figure 1: When seller $v$ joins all platforms (including $i$), buyers' idiosyncratic preferences mean that they spread across platforms 1, 2, and 3 to complete their transaction with the seller (for those that prefer transacting over not transacting).

If the seller quits platform $i$ (in response to $i$'s higher seller fee), it faces the following trade-off. First, it will divert some of the buyers who initially use platform $i$ to switch to platforms $j \in \mathbf{N}_{-i}$. This raises the transactions on each of these platforms by

$$B_j^{(\mathbf{N}_{-i})} - B_j^{(\mathbf{N})} > 0$$

and allows the seller to enjoy the lower fee for each of these diverted transactions. Second, given that the market is not fully covered, some buyers will stop transacting (choosing the outside option) instead of transacting through other platforms. The seller will lose access to these buyers' transactions. To proceed, we define the following notation:

**Definition 1** *For each given profile of buyer fees* $(p_1^b, ..., p_n^b)$*, buyers' loyalty to platform* $i \in \mathbf{N}$ *is defined as*

$$\sigma_i \equiv \frac{B_0^{(\mathbf{N}_{-i})} - B_0^{(\mathbf{N})}}{B_i^{(\mathbf{N})}} \in (0, 1). \tag{2}$$

*where* $B_0^{(\Theta^v)} = \Pr(\epsilon_0 \geq \max_{j \in \Theta^v} \{\epsilon_j - p_j^b\})$.

Here $\sigma_i$ measures buyer loyalty (in their transaction behavior) in the sense that it indicates the fraction of buyers who stop transacting when platform $i$ ceases to be available for transactions with a given seller.[10] Note that $\sigma_i$ is relevant even though buyers are multihoming on

[10] Our definition of loyalty is related to the concept of the aggregate diversion ratio (ADR) by Katz and Shapiro (2003), which measures the fraction of the total sales lost by a firm $i$ (when its price rises by a small percentage amount) that are captured by all of the competing firms $j \neq i$. The inverse of buyer loyalty, $1 - \sigma_i$, is analogous to ADR in the sense that $1 - \sigma_i$ similarly measures the fraction of buyers who switch to the competing platforms. The key distinction is that in the definition of $1 - \sigma_i$ it is as if buyers face an infinitely higher price to use platform $i$ rather than the small percentage increase used to define ADR, reflecting that sellers delist from platform $i$ in our case.

all platforms. Buyers can consider all options but may still have strong preferences towards using certain platforms to complete transactions, and be very reluctant to use others. Then, the change in seller $v$'s net surplus from quitting the most expensive platform $i$ can be written as

$$\underbrace{(p_i^s - \hat{p}^s)(1-\sigma_i)B_i^{(\mathbf{N})}}_{\text{gain from diverting buyers to cheaper platforms}} - \underbrace{(v - p_i^s)\,\sigma_i B_i^{(\mathbf{N})}}_{\text{foregone transactions}}, \tag{3}$$

where the two components indicate the trade-off between saving on fees and attracting fewer transactions, as illustrated in Figure 2 below.

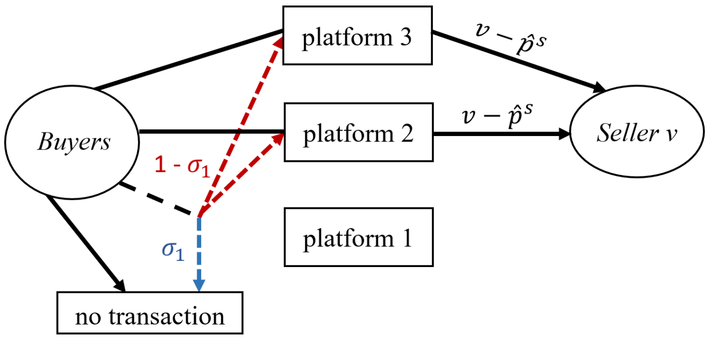


Figure 2: When seller $v$ quits platform 1, a fraction $1-\sigma_1$ of buyers who were using platform 1 to transact with the seller would switch to use platforms 2 and 3, while the remaining fraction $\sigma_1$ of buyers stop transacting with the seller altogether.

Solving for the indifference condition associated with (3) yields seller participation decisions:

**Lemma 1** *Suppose $p_i^s \geq \hat{p}^s$. There exists a threshold*

$$\hat{v} = \frac{p_i^s - \hat{p}^s}{\sigma_i} + \hat{p}^s$$

*such that a seller of type $v$ joins no platform if $v < \hat{p}^s$, joins all platforms $j \neq i$ if $\hat{p}^s \leq v < \hat{v}$, and joins all platforms including $i$ if $v \geq \hat{v}$.*

Notice that the term $\sigma_i$ plays a significant role in understanding how sellers react to changes in seller fees set by platforms. From (2), we can interpret the term $\sigma_i$ as measuring the extent to which buyers *cannot be diverted in their transaction decisions.*

If $\sigma_i$ is close to one, it means that platform $i$ is not substitutable by other platforms $j \neq i$ for buyers who prefer $i$ the most. Whenever platform $i$ is not available for transactions with a particular seller, many existing buyers who have joined platform $i$ will simply stop transacting with the seller (even though they have the option to transact with the seller through other platforms). In other words, it is hard for each seller to divert buyers to transact through the platform that the seller prefers. Thus, a large $\sigma_i$ means that sellers are less likely to quit platform $i$ following an increase in $p_i^s$, i.e.,

$$\frac{d\hat{v}}{dp_i^s} = \frac{1}{\sigma_i}$$

is small. If $\sigma_i$ is close to zero instead, buyers on platform $i$ are unlikely to stop transacting, so that it is easy for sellers to divert the buyers' choice of platform for completing transactions. In this case, sellers are more likely to quit platform $i$ following an increase in $p_i^s$, i.e., $\frac{d\hat{v}}{dp_i^s}$ is large.

☐ **Volume of transactions.** Recall that each buyer-seller pair corresponds to one potential transaction. To derive the number of transactions $Q_i$ facilitated by the deviating platform $i$, we count the number of buyers who use $i$ to transact with each seller $v$, and sum this up over the set of all sellers that are available on platform $i$. That is, for all $p_i^b \neq \hat{p}^b$ and $p_i^s \geq \hat{p}^s$, we have

$$\begin{aligned} Q_i\left(\mathbf{p}_i;\hat{\mathbf{p}}\right)|_{p_i^s \geq \hat{p}^s} &= \int_{\{v|i\in\mathbf{\Theta}_v\}} B_i^{(\mathbf{\Theta}_v)} dG(v) \\ &= \left(1-G\left(\hat{v}\right)\right) B_i^{(\mathbf{N})}, \end{aligned} \tag{4}$$

where the second equality is due to Lemma 1. In short, platform $i$'s increase in $p_i^s$ trades off between fewer sellers participating (hence fewer transactions) and a higher fee.

We relegate the demand derivation for the more complicated case of $p_i^s < \hat{p}^s$ to Section A.1 of the Appendix and provide a sketch of the analysis here. When platform $i$ sets $p_i^s < \hat{p}^s$, each seller faces a trade-off that is similar to, but the reverse of (3): by quitting the more expensive platforms $j \neq i$, the seller gains from diverting some buyers to use the cheaper platform $i$ but loses transactions with buyers who switch to the outside option. Thus, in response to $p_i^s < \hat{p}^s$, some sellers join strictly less than $n$ platforms (while still joining platform $i$). Specifically, there exists a sequence of cutoffs $\hat{v}_n \geq \hat{v}_{n-1} \geq ... \geq \hat{v}_1 = p_i^s$ such that a seller joins $m$ platforms (including $i$) if and only if $v \in [\hat{v}_m, \hat{v}_{m+1})$, where $m \in \{1, 2..., n-1\}$. When buyers want to transact with sellers that join less than $n$ platforms, they have fewer platforms to choose from, which implies that they are more likely to use platform $i$ (if they transact at all). As such, platform $i$'s decrease in $p_i^s$ trades off between inducing sellers to multihome on fewer other platforms (which results in more transactions) and earning a lower fee.

Imposing specific distribution functions allows us to express the volume of transaction (for $p_i^s < \hat{p}^s$) in a simpler form. Suppose $F$ and $F_0$ correspond to the Gumbel distributions with a common scale parameter $\mu$ and location parameters $\beta$ and $\beta_0$ respectively (and normalize $\beta = 0$ without loss of generality). In this case, buyer quasi-demand in (1) follows the standard logit form widely used in the industrial organization literature:

$$B_i^{(\Theta^v)} = \frac{\exp\left\{-p_i^b/\mu\right\}}{\exp\left\{\beta_0/\mu\right\} + \sum_{j\in\Theta^v} \exp\left\{-p_j^b/\mu\right\}}. \tag{5}$$

In Section A.1 of the Appendix, we show that (5) implies $\hat{v}_m = (\hat{p}^s - p_i^s)\exp\left\{-p_i^b/\mu\right\} + \hat{p}^s$ is independent of $m$ for all $m \geq 2$. As such, each seller either joins all platforms including $i$, joins only platform $i$, or joins no platforms, and so

$$Q_i\left(\mathbf{p}_i;\hat{\mathbf{p}}\right)|_{p_i^s < \hat{p}^s} = \left(1-G\left(\hat{v}_m\right)\right) B_i^{(\mathbf{N})} + \left(G\left(\hat{v}_m\right) - G(p_i^s)\right) B_i^{(\{i\})}.$$

### 3.2 Equilibrium fees

We now characterize the equilibrium in the first stage. In what follows, we assume that platform $i$'s profit function

$$\Pi_i = \left(p_i^b + p_i^s - c\right) Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right)$$

is quasi-concave in $(p_i^s, p_i^b)$.[11] In Section A.6 of the Appendix, we show that a sufficient condition for quasi-concavity is that buyer quasi-demand takes the logit form (5) and that $G$ is linear.[12]

For any arbitrarily given (symmetric) buyer fee $p^b$, we define the *buyer inverse semi-elasticity* as

$$X(p^b; n) \equiv \left| \frac{B_i^{(\mathbf{N})}}{\partial B_i^{(\mathbf{N})} / \partial p_i^b} \right|_{p_i^b = p_j^b = p^b}, \tag{6}$$

which is a standard index that measures the competitive markup a firm can extract from buyers in a given equilibrium with $n$ competing firms (Perloff and Salop, 1985). Similarly, we define the buyer loyalty index as the symmetric counterpart of (2):

$$\sigma(p^b; n) \equiv \sigma_i|_{p_i^b = p_j^b = p^b} \in (0, 1) \tag{7}$$

which captures buyers' tendency to stop transacting when their most-preferred platform ceases to be available for transactions, i.e., how difficult it is for sellers to divert buyers' transactions across platforms.

The standard first-order condition for optimal pricing leads to the following equilibrium.[13]

**Proposition 1** *A pure symmetric pricing equilibrium is characterized by all $n$ platforms setting $\hat{\mathbf{p}} = \left(\hat{p}^b, \hat{p}^s\right)$ that solves*

$$\hat{p}^b + \hat{p}^s - c = X(\hat{p}^b; n) = \frac{1 - G(\hat{p}^s)}{g(\hat{p}^s)} \sigma(\hat{p}^b; n). \tag{8}$$

*Moreover, the solution $\hat{\mathbf{p}}$ is unique, i.e., the symmetric equilibrium pinned down by (8) is unique.*

The example below shows that (8) has a closed-form solution for a logit-exponential specification. We will use this example to illustrate several results throughout the paper.

**Example 1** ***(Logit-exponential).*** *Suppose $F$ and $F_0$ correspond to the Gumbel distributions with a common scale parameter $\mu$ and location parameters $\beta$ and $\beta_0$ respectively (and normalize*

[11] Our derivation below focuses on the upward deviation $p_i^s \geq \hat{p}^s$. In Section B of the Online Appendix, we verify that $Q_i$ is always continuous, and that platforms cannot profitably deviate from the equilibrium in (8) by slightly decreasing $p_i^s$. The assumption of quasi-concavity rules out large deviations being profitable.

[12] Beyond the case of the standard logit demand, we numerically check in Section B of the Online Appendix that the profit function is indeed quasi-concave when $F$, $F_0$, and $G$ follow combinations of commonly used distribution functions such as Normal, Exponential, and Gumbel, suggesting that the quasi-concavity of the profit function may indeed hold quite generally.

[13] An assumption implicit in this equilibrium characterization is that the distribution supports are large enough such that platforms' optimal prices are interior solutions.

*$\beta = 0$). Then, expressions (6) and (7) become*

$$X(p^b; n) = \mu \left( \frac{\exp\{\beta_0/\mu\} + n \exp\{-p^b/\mu\}}{\exp\{\beta_0/\mu\} + (n-1)\exp\{-p^b/\mu\}} \right) \tag{9}$$
$$\sigma(p^b; n) = \frac{\exp\{\beta_0/\mu\}}{\exp\{\beta_0/\mu\} + (n-1)\exp\{-p^b/\mu\}}.$$

*Suppose further that $G$ corresponds to the exponential distribution with inverse scale parameter $\theta > 0$. Assuming that $\theta\mu < 1$ and that $\beta_0$ is not too small so that boundary constraints do not bind,[14] the solution to (8) is*

$$\hat{p}^b = \mu \ln\left(\frac{n\theta\mu}{1-\theta\mu}\right) - \beta_0 \quad \textit{and} \quad \hat{p}^s = c + \frac{n\mu}{n-(1-\theta\mu)} - \hat{p}^b. \tag{10}$$

*At the equilibrium prices, $X(\hat{p}^b; n) = \frac{1}{\theta}\sigma(\hat{p}^b; n) = \frac{n\mu}{n-(1-\theta\mu)}$.*

Condition (8) can be intuitively understood as the intersection of equilibrium conditions for the competition in the buyer-side and seller-side markets. To see this, we first denote $P^b(p^s)$ as a function defined implicitly by $p^b$ that solves

$$p^b = \underbrace{c}_{\text{cost}} + \underbrace{X(p^b)}_{\text{market power over buyers}} - \underbrace{p^s}_{\text{cross-subsidy due to revenue from sellers}} \tag{11}$$

and denote $P^s(p^b)$ as a function defined implicitly by $p^s$ that solves

$$p^s = \underbrace{c}_{\text{cost}} + \underbrace{\frac{1-G(p^s)}{g(p^s)}\sigma(p^b; n)}_{\text{market power over sellers}} - \underbrace{p^b}_{\text{cross-subsidy due to revenue from buyers}}. \tag{12}$$

For each arbitrarily given (common) seller-side fee, $P^b(p^s)$ defined by (11) can be understood as a curve that maps out the "one-sided" equilibrium buyer fee. Likewise, $P^s(p^b)$ is a curve that maps out the "one-sided" equilibrium seller fee for each arbitrarily given buyer fee. Then, the equilibrium (8) is simply the unique intersection of the $P^s(p^b)$ and $P^b(p^s)$ curves, each representing the equilibrium condition on each side of the market.

This reinterpretation of the equilibrium provides an intuitive way to understand Proposition 1. Expression (11) represents the standard oligopoly pricing equilibrium (in setting the buyer fee) with competitive markup $X$, except that the price is adjusted downward (upward) because the platform is compensated by the positive (negative) seller fee collected from each transaction. Expression (12) is the sum of cost, adjusted by the buyer fee, plus the standard monopoly pricing markup $\frac{1-G(P^s)}{g(P^s)}$ that is discounted by the buyer loyalty index, $\sigma < 1$. The discount reflects that platform market power over sellers increases when it becomes harder for sellers to divert buyers to transact through different platforms. We discuss the implications of this equilibrium condition in the next subsection.

[14] Specifically, we require $\hat{p}^s \geq 0$ given the exponential distribution has a non-negative support. More generally, we can introduce an additive shifter to the exponential distribution to allow the interior solution of $\hat{p}^s$ to take negative values. See, e.g, the discussion in Section 5.3.

### 3.3 Discussion

□ **Comparison with pure membership models.** The pricing equations (11) and (12) in our pure transaction pricing model somewhat resemble those obtained in the pure membership pricing models of Armstrong (2006), Belleflamme and Peitz (2019a), and Tan and Zhou (2021) in the sense that there is a "*cross-subsidy adjustment*" on each side due to the two-sidedness of the market. However, there is a key conceptual difference in terms of how the subsidy adjustment arises in these two classes of models.

To see this, consider the determination of the seller fee (a similar logic applies for the buyer fee). In membership pricing models, the subsidy adjustment reflects the cross-group membership externality, whereby an increase in seller participation raises buyers' willingness to pay for platform membership. In cases where the cross-group externality is negative, e.g., if we replace "sellers" with "advertisers", then the subsidy adjustment would have a negative sign. However, in transaction pricing models, the cross-group externality is irrelevant in the determination of transaction fees. Instead, any subsidy adjustment in the transaction fees reflects the "*usage externality*" emphasized by Rochet and Tirole (2003, 2006) — an additional transaction caused by an increase in seller participation has a cost of $c$ but generates an offsetting subsidy of $p^b$, so this subsidy should be taken into account in setting the price to sellers, and vice-versa. Moreover, the sign of the cross-subsidy adjustment is primarily determined by the nature of the value distributions of buyers and sellers.

□ **Membership fee component.** Consistent with our motivating examples, we have focused on platforms charging per-transaction fees. A natural extension is to allow platforms to charge two-part tariffs that include membership fees (in addition to transaction fees). In Section C of the Online Appendix, we focus on the case of logit buyer quasi-demand and show that Proposition 1 remains an equilibrium in this extended setup if buyer and seller beliefs are such that buyers and sellers coordinate on not participating on platforms that charge strictly positive membership fees (to either buyers or sellers) whenever this is an equilibrium in the resulting subgame.[15] Such beliefs capture the idea that facing competition with other platforms, membership fees make it more difficult for a platform to attract users, perhaps explaining why few platforms charge such fees in practice.[16]

□ **Significance of index $\sigma$.** In our model of usage externality, multihoming in participation does not automatically imply certain market outcomes because one has to take into account users' transaction behavior, which is summarized by $\sigma$. In our setup, all buyers are free to join all platforms and so they all fully multihome in terms of their participation. Yet, if $\sigma \to 1$, buyers' transaction pattern would exhibit a strong tendency of being loyal to a single

[15] The key step is to show that no platform can profitably deviate from the equilibrium in Proposition 1 by setting negative membership fees. The intuition of the result is that negative membership fees are a less effective instrument to expand the deviating platform's transaction volume compared to lowering its transaction fees.

[16] If we allow for other types of beliefs, then a platform may be able to profitably deviate by significantly lowering its buyer transaction fees while adjusting (positive) buyer membership fees to keep the platform's buyer-side revenue unchanged. This maneuver makes it harder for sellers to divert buyers' transaction away from the deviating platform (as the buyer membership fee is sunk at the point of transaction), allowing the platform to charge a higher seller transaction fee. Consequently, characterizing the pricing equilibrium in this case is difficult and goes beyond the scope of the current paper. This complexity arises due to two-sided multihoming, which is a feature absent in previous two-sided market models with two-part tariffs, e.g., Reisinger (2014).

platform, in the sense that they are likely to stop transacting whenever their most-preferred platform ceases to be available even though they have joined other alternative platforms. The resulting buyers' singlehoming-type behavior in transactions generate the familiar "competitive bottleneck" outcome (Armstrong, 2006; Armstrong and Wright, 2007) despite the multihoming in participation. That is, buyers behave as if they only participate on a single platform. Platforms exert monopoly power over sellers (letting $\sigma \to 1$ in (12)) and compete intensely for buyers. Such a competitive bottleneck disappears in the opposite case of $\sigma \to 0$, whereby buyers' transaction pattern reflects a strong willingness to switch to other platforms whenever any particular platform is no longer available (this case is only possible if $n \geq 2$).

The index $\sigma$ helps to make clear the nature of the platforms' market power over sellers, whereby the elasticity of seller participation is closely related to buyers' behavior (even if we ignore any cross-subsidization effect). In the extreme case where $\sigma \to 0$, each buyer necessarily purchases one product from each seller in equilibrium, and buyers are willing to do so through any of the $n$ platforms. In this case, platforms have zero market power over sellers (recall that sellers view platforms as homogenous) because if one platform tries to charge more, sellers can always divert sales through one of the alternative (cheaper) platforms without losing any transactions. Thus, in our framework, each platform's market power over sellers stems primarily from the possibility that sellers may lose access to some buyers when they delist from the platform.

Finally, in the special case of $n = 2$ and the $F_0$ is a degenerate distribution at $\epsilon_0 = 0$ (i.e., each buyer obtains the same surplus from all potential transactions on the same platform), (8) recovers the duopoly equilibrium of Rochet and Tirole (2003, Proposition 3). We show that their pricing formula generalizes to oligopolistic platforms with transaction-specific buyer surpluses. A key difference is we express the formula in terms of the underlying distributions of buyers' and sellers' valuations over interaction benefits. This allows us to have a micro-founded understanding of the nature of the index $\sigma$, and the comparative statics of the factors which drive it, which we explore in Sections 4-6.[17]

# 4 Platform entry

## 4.1 Fee implications

In this section we explore how increased platform competition (i.e. entry) affects the platforms' equilibrium total fee and fee structure. We start by examining how an increase in $n$ affects: (i) the competition for sellers captured by (12), which depends on $\sigma(p^b; n)$; and (ii) the competition for buyers captured by (11), which depends on $X(p^b; n)$. Then, we combine these to obtain the overall effect of an increase in $n$.

We first state the following lemma, which we have proven in the proof of Proposition 1.

**Lemma 2** *For each given $p^b$:*

- *The buyer loyalty index $\sigma(p^b; n)$ defined in (7) is decreasing in $n$.*

[17] Rochet and Tirole (2003) label the term $\sigma$ as the buyer "singlehoming index". We use the word "buyer loyalty "to highlight that $\sigma$ is not necessarily tied to the homing behavior of buyers.

- *The buyer inverse semi-elasticity $X(p^b; n)$ defined in (6) is decreasing in $n$.*

☐ **Intensified seller-side competition ($\partial\sigma/\partial n < 0$).** When $n$ increases, the first part of Lemma 2 implies that buyer loyalty decreases because the platforms become more substitutable. Sellers find it easier to divert buyers to transact through different platforms, so that they are more likely to quit platforms that charge high seller fees. Thus, the seller-side competition, as captured by the seller-side curve $P^s\left(p^b\right)$ in (12), becomes more intense when $n$ increases. We write $P^s(p^b; n)$ to make explicit this dependency on $n$. Graphically, when the number of platforms increases from $n_1$ to $n_2$, the seller-side curve shifts downward from the solid line $P^s(p^b; n_1)$ to the dotted line $P^s(p^b; n_2)$, as shown in Figure 3.

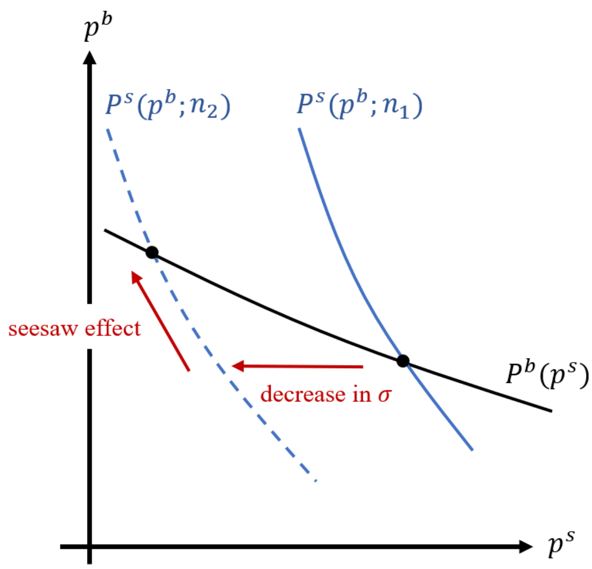


Figure 3: Intensified competition for sellers ($n_2 > n_1$)

All else equal, the shift in the seller-side curve has two effects: (i) it decreases the equilibrium seller fee *directly*; and (ii) it increases the equilibrium buyer fee *indirectly* through the movement along the $P^b(p^s)$ curve. The latter effect reflects the well-known *seesaw effect* in the two-sided market literature (Rochet and Tirole, 2006): when the seller fee decreases by $\Delta p^s < 0$ (due to intensified competition for sellers), the effective marginal cost of servicing buyers, $c - p^s$, increases. It becomes less valuable for platforms to attract transactions by buyers so that the buyer fee increases by

$$\Delta p^b \approx \frac{\partial P^b(p^s)}{\partial p^s}\Delta p^s = \underbrace{\frac{-\Delta p^s}{1 - \frac{\partial}{\partial p^s}\frac{1-G(p^s)}{g(p^s)}}}_{>0 \text{ (see-saw effect)}}. \tag{13}$$

Nonetheless, log-concavity of $1 - G$ implies an incomplete pass-through property, meaning that (13) is smaller than $\Delta p^b$ in magnitude and so the equilibrium total fee decreases.

☐ **Intensified buyer-side competition ($\partial X/\partial n < 0$).** The second part of Lemma 2 implies that the equilibrium buyer-side competitive markup, $X$, decreases with $n$. This reflects the standard intuition that platforms are more substitutable for buyers when buyers have more platforms to choose from.[18] Consequently, the buyer-side competition, as captured by the buyer-side curve $P^b(p^s)$ in (11) becomes more intense. Graphically, when the number of platforms increases from $n_1$ to $n_2$, the buyer-side curve shifts downward from the solid line $P^b(p^s; n_1)$ to the dotted line $P^b(p^s; n_2)$ in Figure 4.

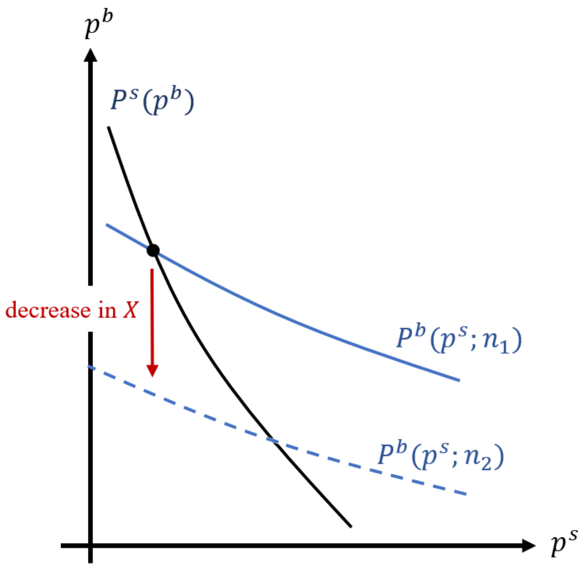


Figure 4: Intensified competition for buyers ($n_2 > n_1$)

All else equal, the shift in the buyer-side curve has a *direct* effect of decreasing the equilibrium buyer fee, but its *indirect* effect on the equilibrium seller fee can be ambiguous, depending on the shape of $P^s(p^b)$ curve. Specifically, when the buyer fee decreases by $\Delta p^b < 0$ (due to intensified competition for buyers), the seller fee changes by

$$\begin{aligned} \Delta p^s &\approx \frac{\partial P^s(p^b)}{\partial p^b}\Delta p^b \\ &= \underbrace{\frac{-\Delta p^b}{1-\partial X/\partial p^s}}_{>0 \text{ (see-saw effect)}} + \underbrace{\frac{\partial \sigma/\partial p^b}{1-\partial X/\partial p^s}\Delta p^b}_{<0 \text{ (decreased loyalty due to downward pressure on } X)} \end{aligned} \tag{14}$$

A lower buyer fee means it becomes less valuable to attract sellers to participate and so the seller fee increases, as represented by the positive see-saw effect in (14). At the same time, the downward pressure on buyer fee reduces buyer loyalty ($\partial\sigma/\partial p^b > 0$) because when the outside option becomes relatively less attractive for buyers, it is easier for each seller to divert buyers' transactions. This channel, as represented by the negative second term in (14), exerts a downward pressure on the seller fee. Nonetheless, log-concavity of density functions $f$ and $f_0$ implies an incomplete pass-through property, meaning that (14) is smaller than $\Delta p^b$ in magnitude and so the equilibrium total fee must decrease.

☐ **Overall effect of competition.** In sum, an increase in $n$ affects the equilibrium fees via two separate effects: *intensified competition for sellers* ($\partial\sigma/\partial n < 0$) and *intensified competition for buyers* ($\partial X/\partial n \leq 0$). These two effects lead to an unambiguous decrease in the equilibrium

[18] This is an extension of the result by Zhou (2017) which considers the case where $F_0$ is a degenerate distribution at $\epsilon_0 = 0$.

total fee, but the corresponding changes in $\hat{p}^b$ and $\hat{p}^s$ are ambiguous in general. In particular, the two-sidedness of the market means that the initial decrease in $\hat{p}^b$ and $\hat{p}^s$ (due to intensified competition for sellers and for buyers) has to be adjusted by the additional effects noted in (13) and (14).

In the proof of the next proposition, we show that the overall effects of $n$ on $\hat{p}^b$ and $\hat{p}^s$ can be decomposed as:

$$\frac{d\hat{p}^b}{dn} = (\textit{initial decrease in } X \textit{ due to } n) \text{ - } (\textit{net see-saw effect})$$

and

$$\begin{aligned}\frac{d\hat{p}^s}{dn} \quad = \quad & (\textit{initial decrease in } \sigma \textit{ due to } n) + (\textit{net see-saw effect}) \\ & + (\textit{decrease in } \sigma \textit{ due to downward pressure on } X),\end{aligned}$$

where the net see-saw effect is negative if and only if the shift in the seller curve (in Figure 3) dominates the shift in the buyer curve (in Figure 4), i.e.,

$$\left| \frac{\partial \sigma / \partial n}{\sigma} \right| > \left| \frac{\partial X / \partial n}{X} \right| \tag{15}$$

near the initial equilibrium point $\hat{p}^b$. Property (15) says that buyer loyalty index $\sigma$ is more elastic with respect to changes in $n$ compared to the buyer-side competitive markup $X$. For instance, in our logit-exponential example, a simple calculation shows $\left| \frac{\partial \sigma / \partial n}{\sigma} \right| - \left| \frac{\partial X / \partial n}{X} \right| = B_i^{(\mathbf{N})} = \frac{1}{n} (1 - \theta \mu) > 0$.

We find that a sufficient condition for (15) is that the density function $f$ is weakly decreasing. In this case, the effect of intensified competition for sellers dominates, which leads to the following formal result that holds for all $n \geq 1$.

**Proposition 2** *(Increased platform competition) In the equilibrium characterized by Proposition 1, an increase in $n$ (i.e. platform entry) decreases the total fee $\hat{p}^s + \hat{p}^b$. Furthermore, an increase in $n$ decreases $\hat{p}^s$ if (15) holds, and increases $\hat{p}^b$ if density functions $f$ and $g$ are weakly decreasing.*

A novel implication of Proposition 2 is that, when there is two-sided multihoming, increased platform competition tends to shift the fee structure in favor of sellers. The property (15) plays a key role in establishing this result. Interestingly, the property depends only on the preference distribution of the buyers, i.e., $F$ and $F_0$. The sufficient condition of weakly decreasing density function $f$ is satisfied by some commonly used distributions such as the uniform distribution, the exponential distribution, the power law distribution, and the generalized Pareto distribution (for a certain range of parameter values). By way of comparison, linear demand (analogous to the uniform distribution) has been used in the related literature (e.g., Rochet and Tirole, 2003; Armstrong, 2006; Bakos and Halaburda, 2020).[19]

[19] Weakly decreasing density $f$ is certainly not a necessary condition for (15). In Section A.6 of the Appendix,

Second, property (15) can be understood as follow. Broadly speaking, an increase in $n$ has two effects. First, it makes the platforms more substitutable, which raises the within-market competitive pressure, thus decreasing both $X$ and $\sigma$. Second, it triggers a market expansion effect. This market expansion effect decreases the relative attractiveness of the buyers' outside option and so $\sigma$, thus reinforcing the first effect. In contrast, the market expansion effect increases $X$, thus partially mitigating the first effect on $X$. As a result, $X$ tends to be less elastic towards changes in $n$ than is $\sigma$, explaining why property (15) holds under fairly general distributional assumptions.

Finally, weakly decreasing densities $f$ and $g$ guarantee that the (negative) net see-saw effect has a relatively large magnitude which dominates the initial decrease in $X$ in (13), and so $\hat{p}^b$ unambiguously increases with $n$. Notice that weakly decreasing densities are not necessary conditions. In our logit-exponential example, it is clear from (10) that $\hat{p}^b$ increases with $n$ while $\hat{p}^s$ decreases with $n$ even though the Gumbel distribution does not have a monotone decreasing density. Nonetheless, in more general cases, the net see-saw effect could be weak so that $\hat{p}^b$ decreases (or is non-monotone) when $n$ increases. This is the case when, for example, the density functions are increasing with sufficiently steep gradients.

### 4.2 Surplus implications

To discuss the surplus implications of platform competition, let us define expected per-transaction surplus for buyers and sellers as

$$\begin{aligned} V^b(\hat{p}^b) &\equiv \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \max\left\{\epsilon - \hat{p}^b, \epsilon_0\right\} dF^n(\epsilon)\, dF_0(\epsilon_0) \\ V^s(\hat{p}^s) &\equiv \int_{\hat{p}^s}^{\bar{v}} (v - \hat{p}^s)\, dG(v)\,. \end{aligned}$$

Then, buyer and seller surpluses are

$$BS = V^b(\hat{p}^b)\left(1 - G(\hat{p}^s)\right) \qquad \text{and} \qquad SS = V^s(\hat{p}^s) \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} 1 - F(\epsilon_0 + \hat{p}^b)^n dF_0(\epsilon_0)\,.$$

Changes in fees can affect the surplus of each user side in two ways: (i) a direct effect through the per-transaction surplus; and (ii) a transaction volume effect. Thus, even if platform competition increases $\hat{p}^b$ (hence a lower $V^b$), buyer surplus would still increase if the competition decreases $\hat{p}^s$ by a sufficiently large magnitude (more sellers available for transactions). Likewise, a decrease in $\hat{p}^s$ does not necessarily increase seller surplus if the competition increases $\hat{p}^b$ (fewer transactions by buyers).

Consequently, the surplus implications of competition are ambiguous in general even if we can pin down the price implications (i.e., when the conditions in Proposition 2 hold). Nonetheless, in our logit-exponential example, we have the following formal result:[20]

we show that (15) holds as long as $F$ and $F_0$ correspond to Gumbel distribution, which does not have monotone decreasing density. We also numerically verify that property (15) holds when $F$ and $F_0$ follow normal, gamma, and power function distributions (provided the second order conditions for equilibrium are satisfied). These suggest that (15) is indeed true quite generally whenever second order conditions hold.

[20] See Section A.7 of the Appendix for details.

**Corollary 1** *In the logit-exponential example, an increase in $n$ (i.e. platform entry) increases buyer surplus, seller surplus, and total surplus.*

One interpretation of this result is that in the logit-exponential setting with all buyers multihoming, the decrease in $\hat{p}^s$ due to entry is significantly larger than the increase in $\hat{p}^b$, so that: (i) on the seller side the positive effect through the per-transaction surplus dominates the negative effect of transacting with fewer buyers; (ii) on the buyer side, the negative effect through the per-transaction surplus is dominated by the positive effect of transacting with more sellers. The result illustrates the possibility that looking at the effect of entry (or mergers) on prices charged to customers on one side alone can be a misleading guide to whether customers on that side are better or worse off, and the importance of taking into account feedback effects via the other side.

# 5 Multihoming behavior of buyers

Recall that in our benchmark setting, each buyer faces zero joining cost regardless of the number of platforms that he has already joined, and so all buyers multihome on all platforms in the equilibrium. In Sections 5.1 and 5.2, we explore how the cost of buyers multihoming affects (i) the market equilibrium; and (ii) the effects of platform entry. In Sections 5.3 and 5.4, we discuss the implications of these results in the specific applications of ride-hailing and payment card platforms. To keep the exposition brief, we focus on presenting the main insights in this section and relegate the equilibrium analysis to Section D of the Online Appendix.

## 5.1 A model of buyer partial multihoming

We allow some buyers to singlehome by extending the model in Section 2 as follows. Suppose that buyers obtain some stand-alone participation benefit (can be zero) from joining at least one platform and then incur a cost $\psi$ (or a benefit if $\psi < 0$) for each additional platform joined. Buyers have heterogenous $\psi$, distributed according to some CDF $F_\psi$.

Denote $1 - \lambda \equiv 1 - F_\psi(0)$ as the fraction of buyers with $\psi > 0$. Provided that these buyers expect each seller either multihomes on all platforms or joins no platform (which we will show to be true in equilibrium), they do not expect to gain additional access to sellers by joining more than one platform. Hence, these buyers join at most one platform in the equilibrium, i.e., they singlehome. The remaining fraction $\lambda \equiv F_\psi(0)$ of buyers have $\psi \leq 0$ (i.e. there is some non-negative stand-alone benefit from joining additional platforms), so that these buyers multihome on all platforms in the equilibrium, as in the benchmark model. Notice that $\lambda = 1$ corresponds to our benchmark setting.

To keep the exposition as simple as possible for this application, we assume that singlehoming buyers observe only buyer fees and not seller fees.[21] These buyers hold passive beliefs (Hart

[21] Recall in our benchmark setting with $\lambda = 1$, whether buyers observe seller fees or not does not affect the analysis. Consistent with our assumption here, Janssen and Shelegia (2015) note that vertical arrangements between sellers and platforms are typically confidential, and so are not observed by buyers. Hagiu and Hałaburda (2014) and Belleflamme and Peitz (2019b) have analyzed the implications of this informational assumption for pricing in two-sided markets.

and Tirole, 1990) on the unobserved seller fees, meaning they believe seller fees are equal to the equilibrium levels whenever they observe an off-equilibrium buyer fee. We consider the alternative assumption of singlehoming buyers observing the seller side fees in Section D.1 of the Online Appendix, focusing on the polar cases of $\lambda \to 0$ and $\lambda \to 1$.[22]

The derivation for this partial-multihoming model largely follows those in Section 3. We first note that the competition on the buyer side is independent of $\lambda$ in the equilibrium. To see this, consider a singlehoming buyer's participation decision. The buyer will join the platform that yields the highest expected utility, taking into account the number of sellers on each of the platforms. Since the buyer does not observe seller fees and holds passive beliefs, he takes $p_i^s$ as fixed at the equilibrium level $\hat{p}^s$, which is the same across all platforms. Given this, the singlehoming buyer expects the same set of sellers on each platform in the equilibrium and he will join only the platform that gives the highest per-transaction surplus. That is, he joins platform $i$ if and only if $\epsilon_i - p_i^b \geq \max_{j\in\mathbf{N}}\{\epsilon_j - p_j^b\}$. After joining platform $i$, the buyer uses it for a transaction (with each seller) if $b_0 + \epsilon_i - p_i^b > 0$, or equivalently if $\epsilon_i - p_i^b \geq \epsilon_0$. Therefore, the total mass of singlehoming buyers who use platform $i$ for transactions is

$$\Pr\left(\epsilon_i - p_i^b \geq \max_{j\in\mathbf{N}}\left\{\epsilon_j - p_j^b, \epsilon_0\right\}\right),$$

which is exactly (1) whenever $\Theta^v = \mathbf{N}$.

However, the presence of some singlehoming buyers means sellers, whenever they quit one of the platforms, divert less buyers to other platforms for transactions, i.e., the transaction loyalty index increases when $\lambda$ decreases. This allows platforms to exercise greater market power over sellers. Specifically, we can define the counterpart of (7) for this environment:

$$\sigma_\lambda(p^b; n) \equiv \lambda\sigma(p^b; n) + 1 - \lambda. \tag{16}$$

Notice that $\sigma_\lambda$ is decreasing in $\lambda$. If $\lambda = 0$ then $\sigma_\lambda = 1$, i.e., buyers cannot be diverted to use other platforms for transactions because all of them join only one platform. If $\lambda = 1$ then $\sigma_\lambda < 1$ defined here corresponds to the benchmark definition in (7). Thus, $\sigma_\lambda$ relates buyer transaction behavior with their participation homing behavior.

In this environment, a pure symmetric pricing equilibrium can be characterized by all platforms choosing $\hat{\mathbf{p}} = \left(\hat{p}^b, \hat{p}^s\right)$ that uniquely solves

$$\hat{p}^b + \hat{p}^s - c = X(\hat{p}^b; n) = \frac{1 - G\left(\hat{p}^s\right)}{g\left(\hat{p}^s\right)}\sigma_\lambda(\hat{p}^b; n). \tag{17}$$

Given that an increase in $\lambda$ always decreases $\sigma_\lambda$ but does not affect $X$, the logic in Section 4 immediately implies the following result:

[22] The key distinction in this case is that each platform can attract buyers with $\psi > 0$ by lowering its seller fee (so that these buyers can transact with more sellers when they singlehome on the platform), which leads to a new multiplicative coefficient that discounts the platform market power over sellers. Assuming logit buyer quasi-demand, we show that if platforms are sufficiently differentiated from buyers' perspective, then the qualitative insights in this section remain valid.

**Proposition 3** *(Effect of buyer multihoming) In the equilibrium characterized by (17), a higher fraction of multihoming buyers ($\lambda$) increases $\hat{p}^b$, decreases $\hat{p}^s$, and decreases the total fee $\hat{p}^b + \hat{p}^s$.*

Proposition 3 is analogous to Proposition 5.3 of Rochet and Tirole (2003), but there are three important differences. First, their result focuses on competing associations (each that maximizes the volume of transactions) whereas our result considers proprietary platforms (that maximize profit). Second, our result does not rely on demand linearity and can accommodate an arbitrary number of platforms. Finally, their result is stated in terms of an exogenous increase in $\sigma$ (the "singlehoming index" in their terminology) but does not clarify how does such an exogenous change relates to the homing behaviors of buyers.[23] Our approach provides a microfoundation that links such a change with the fraction of buyers singlehoming due to multihoming costs. This approach also has implications for the competitive bottleneck theory (Armstrong, 2006; Armstrong and Wright, 2007; Belleflamme and Peitz, 2019a) in that when more buyers multihome (an increase in $\lambda$), the competitive bottleneck initially faced by the seller side is reduced.

### 5.2 Interaction with platform entry

We are interested in the interaction between the extent of buyer multihoming ($\lambda$) and the number of platforms that are competing ($n$). Specifically, does homing behaviors of buyer change the implications of platform entry in Proposition 2?

**Proposition 4** *(Increased platform competition) In the equilibrium with partial-multihoming buyers characterized by (17), an increase in $n$ always decreases the total fee.*

1. *If the fraction of buyers multihoming goes to zero ($\lambda \to 0$), an increase in $n$ increases $\hat{p}^s$ and decreases $\hat{p}^b$.*
2. *If the fraction of buyers multihoming goes to one ($\lambda \to 1$) and $f$ and $g$ are weakly decreasing, an increase in $n$ decreases $\hat{p}^s$ and increases $\hat{p}^b$.*

For the logit-exponential example, we prove in Section A.7 of the Appendix a complete characterization which shows both $\hat{p}^s$ and $\hat{p}^b$ decrease with $n$ for intermediate values of $\lambda$.

**Corollary 2** *In the logit-exponential example, there exist cutoffs $\bar{\lambda}_1 = 1 - \theta\mu$ and $\bar{\lambda}_2 \in (0, \bar{\lambda}_1)$ such that:*

1. *If $\lambda < \bar{\lambda}_2$, an increase in $n$ increases $\hat{p}^s$ and decreases $\hat{p}^b$.*
2. *If $\lambda \in [\bar{\lambda}_2, \bar{\lambda}_1]$, an increase in $n$ decreases $\hat{p}^s$ and $\hat{p}^b$.*
3. *If $\lambda > \bar{\lambda}_1$, an increase in $n$ decreases $\hat{p}^s$, and increases $\hat{p}^b$.*

[23] In their primary example of an extended linear Hotelling model, such an exogenous increase in the buyer loyalty index corresponds to an increase in the marginal transportation cost of buyers for distances in the "non-competitive hinterland" of the rival platform while holding constant the transportation cost of all other segments of the Hotelling line.

Proposition 4 highlights a novel finding of our paper: even though increased platform competition always reduces the total fee charged to the two sides, whether it shifts the fee structure in favor of buyers or sellers depends on whether most of the buyers are singlehoming or multihoming. A key step in our proof is showing that $\left|\frac{\partial\sigma_\lambda/\partial n}{\sigma_\lambda/n}\right|$ decreases when $\lambda$ decreases. That is, when more buyers are singlehoming in participation, the loyalty index $\sigma_\lambda$ becomes less responsive towards changes in $n$.

Intuitively, when most of the buyers multihome ($\lambda \to 1$), increased platform competition induces platforms to compete more intensely for sellers, as explained previously (following Proposition 2). However, when most of the buyers singlehome ($\lambda \to 0$), platforms have monopoly power over providing access to their buyers for the multihoming sellers. As such, increased platform competition induces platforms to compete more intensely for buyers rather than for sellers (the normal competitive bottleneck logic). We discuss the economic implication of this result for specific markets in the next two subsections.

### 5.3 Application: ride-hailing platforms

In the context of ride-hailing platforms, for each trip the riders (buyers) enjoy benefits while the drivers (sellers) incur efforts, so that $p^b > 0 > p^s$ in practice. Here, $p^b$ is the fare set by the platforms, the negative value of $p^s$ is the per-ride driver gross earning (or wage). In this context, multihoming riders are those who compare and choose between multiple apps whenever they call for a ride, while singlehoming riders are those who do not do so. To facilitate exposition, we focus on the polar cases of all buyers singlehoming ($\lambda = 0$) and all buyers multihoming ($\lambda = 1$), while noting that the general qualitative insights remain the same for cases between these two extremes ($\lambda \in (0, 1)$).

Figure 5 numerically illustrates this application, assuming that $c = 0.1$, $F$ and $F_0 \sim \textit{Gumbel}$ with scale parameter $\mu = 3$ and location parameters $\beta = 0$ and $\beta_0 = -5$, while $G$ follows the exponential distribution with inverse scale parameter $\theta = x$ and an additively-shifted support of $[-20, \infty)$.

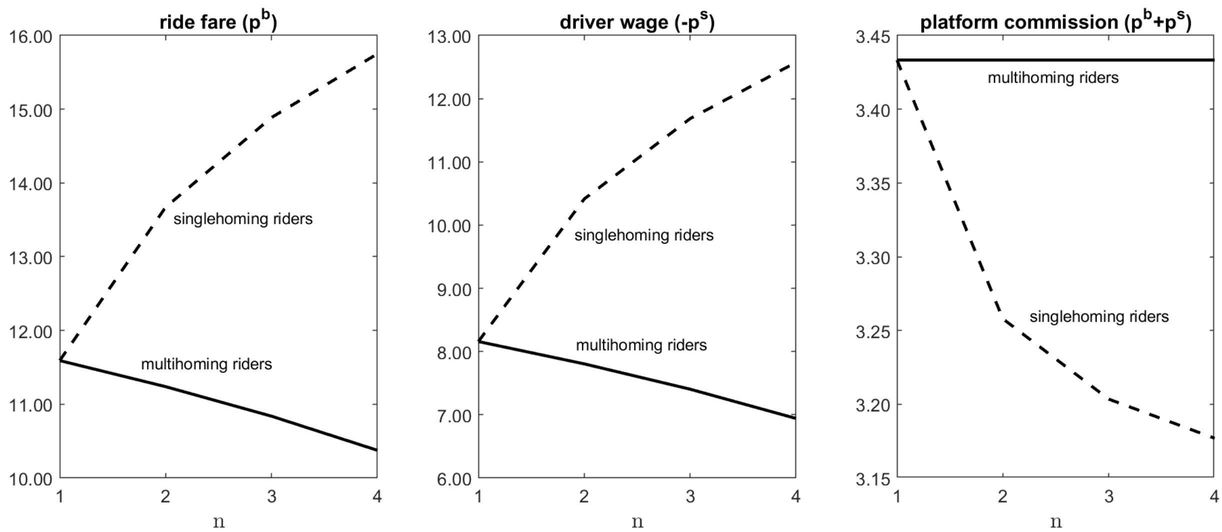


Figure 5: A numerical simulation of a ride-hailing market

☐ **Platform competition.** Buyer multihoming profoundly reverses the dynamics of platform competition. When riders are singlehoming, existing ride-hailing platforms respond to entry by cutting the fare to attract riders, and then reoptimize by offering less to drivers. However, when riders are multihoming, if the incumbent platforms naively continue to respond by cutting fares and driver wages, then some drivers will simply quit the lower-wage incumbents, knowing that they can still access a large portion of riders through other higher-wage platforms. Instead, our analysis suggests that the response in equilibrium would be the reverse: platforms increase wages to attract drivers, and then reoptimize the fare by charging more. The possibility of a fare increase following entry is in contrast to the conventional one-sided logic that high final product prices (in this case, rider fares) are caused by a lack of competition.[24]

☐ **Platform merger and exit.** The industry of ride-hailing services has witnessed several high profile merger cases in recent years, including Didi-Uber in China (2016), Yandex-Uber in Russia (2017), Grab-Uber in South East Asia (2018), and Careem-Uber in Middle East (2019). Notably, each of these mergers has resulted in one of the platforms exiting the market entirely.[25] Based on analyzing what happens when $n$ decreases by one, our analysis suggests that the effect of these mergers on the platform fee structure depends critically on the level of rider-multihoming. This provides an empirical implication: even in the absence of any cost-efficiency gain from the merger, it is possible for such a merger to result in lower fares for riders (if the extent of rider-multihoming is high) or higher earnings for drivers (if the extent of rider-multihoming is low). Regardless of the level of rider-multihoming, however, our model also predicts the total fee charged to the two sides will increase.

## 5.4 Application: payment card platforms

Payment card platforms typically offer card holders (buyers) a variety of card-usage benefits e.g., interest-free periods, cash rebates and loyalty rewards. Platforms then make money by charging transaction fees on merchants (sellers), so that $p^s > 0 > p^b$ in practice. In this context, multihoming cardholders are those who have multiple cards to choose from at the point of transactions, while singlehoming cardholders are those who do not.

☐ **Platform competition and interchange fees.** Policymakers in some jurisdictions, including Australia, Europe, and United Kingdom, have claimed that payment card platforms set interchange fees too high. As summed up by Guthrie and Wright (2007), these authorities appear to view the lack of competition between platforms as a possible cause of high interchange fees. However, Proposition 4 suggests that this view by the authorities is true only when most of the cardholders are multihoming, whereby increasing inter-platform competition indeed helps to reduce the interchange fee paid by the merchant side to the cardholder side. Notably, the reverse view is true when the fraction of singlehoming cardholders is sufficiently large, whereby increasing inter-platform competition drives up the interchange fee instead, which seems to

[24]See also Bryan and Gans (2019) for an investigation of how the multihoming behaviour of riders and drivers affects pricing (and welfare) when there are two competing ride-hailing platforms.

[25]Therefore, these merger cases are different from standard horizontal mergers involving differentiated products, where the merged entity would continue operating both of the original brands so as to maximize their joint profit.

match the empirical evidence better (Rysman and Wright, 2015).[26]

## 6 Value of transactions and user heterogeneity

In this section, we explore how the market equilibrium outcome depends on value of transactions and user heterogeneity. Whenever applicable, we highlight the role of index $\sigma$ in understanding these comparative static exercises. To keep the exposition brief, we relegate details and formal proofs of the propositions to Section E of the Online Appendix.

We extend the model in Section 2 by introducing the following parameters: (i) $\alpha^b$ and $\alpha^s$, which shifts the buyer utility from transacting relative to not transacting; (ii) $\gamma^b > 0$ and $\gamma^s > 0$, which indicates the heterogeneity of buyer preferences and seller preferences. A buyer that uses platform $i$ for a transaction receives net utility[27]

$$(b_0 + \epsilon_i)\gamma^b + \alpha^b - p_i^b$$

while generating to the seller net utility

$$v\gamma^s + \alpha^s - p_i^s.$$

The surplus from the outside option remains fixed at zero and all other specifications remain the same. One interpretation of the comparative static analysis in this section is we are making comparisons within the classes of distributions that are parameterized by location and scale parameters $(\alpha^b, \gamma^b)$ (for the buyer value distributions) and $(\alpha^s, \gamma^s)$ (for the seller value distribution). This parameterization allows us to shift distribution functions in a tractable manner.[28]

This extension is mathematically equivalent to applying a linear transformation to the fees charged by each platform. The equilibrium condition is similar to (8) in the benchmark model:

$$\hat{p}^b + \hat{p}^s - c = \gamma^b X\left(\frac{\hat{p}^b - \alpha^b}{\gamma^b}; n\right) = \frac{1 - G(\frac{\hat{p}^s - \alpha^s}{\gamma^s})}{g(\frac{\hat{p}^s - \alpha^s}{\gamma^s})}\gamma^s \sigma\left(\frac{\hat{p}^b - \alpha^b}{\gamma^b}; n\right), \tag{18}$$

where $X(.; n)$ and $\sigma(.; n)$ are defined in (6) and (7).

**Proposition 5** *(Value of transaction) In the equilibrium (18):*

1. *An increase in the value of transactions for buyers ($\alpha^b$) increases $\hat{p}^b$, decreases $\hat{p}^s$, and increases $\hat{p}^b + \hat{p}^s$.*

[26]This provides another reason why increased platform competition can increase interchange fees separate from the existing explanation of Guthrie and Wright (2007) and Edelman and Wright (2015) which relies on the use of the no-surcharge rule (or price coherence more generally).

[27]Notice that if $\gamma^b = 0$ then all buyers are homogenous. Our formulation follows the standard models of oligopolistic price competition (Anderson et al., 1992; Anderson and Peitz, 2020), which typically impose a common scale parameter to all idiosyncratic components of utility functions.

[28]The parameters in section have natural one-to-one counterparts to the parameters in the logit-exponential example. Specifically, the Gumbel scale parameter ($\mu$) corresponds to $\gamma^b$, Gumbel location parameter of $F_0$ ($\beta_0$) corresponds to $-\alpha^b$, while inverse scale parameter of exponential distribution corresponds to $1/\gamma^s$.

2. *An increase in the value of transactions for sellers ($\alpha^s$) increases $\hat{p}^s$, decreases $\hat{p}^b$, and increases $\hat{p}^b + \hat{p}^s$.*

The first part of Proposition 5 implies that an increase in $\alpha^b$ which increases the value buyers put on transacting with sellers (relative to not transacting) makes competition for sellers more intense. This is intuitive. There are two relevant forces here. First, a higher $\alpha^b$ raises platforms' market power over buyers (they strongly desire transactions now) because transactions can only occur through platforms. Through the see-saw effect, this exerts a downward pressure on $\hat{p}^s$. The second force is more novel: when buyers highly value transacting with sellers, they are willing to use any platform for transaction (as opposed to not transacting). This makes it easier for each seller to, through her participation decision, divert buyers to use the platform that the seller desires. Consequently, platforms have weaker market power over the sellers, which exerts additional downward pressure on $\hat{p}^s$. In the extreme case where $\alpha^b \to \infty$, the outside option becomes irrelevant and $\sigma \to 0$, implying that platforms would have no market power over sellers.

For the second part of Proposition 5, a higher $\alpha^s$ allows platform to charge a higher $\hat{p}^s$ and, through the see-saw effect, exerts a downward pressure on $\hat{p}^b$. In practice, an increase in $\alpha$ may correspond to sellers extracting more surplus from transactions (while leaving buyer surplus unaffected) or an industry-wide increase in the convenience benefit that platforms offer to the sellers. This result extends the insights of Proposition 5.1 of Rochet and Tirole (2003), which focuses on duopolistic competing associations (which maximize volume of transactions instead of profits).

**Proposition 6** *(User heterogeneity) In the equilibrium (18):*

1. *Suppose $\alpha^b$ is small enough, then an increase in the extent of buyer heterogeneity ($\gamma^b$) increases $\hat{p}^b$, decreases $\hat{p}^s$, and increases $\hat{p}^b + \hat{p}^s$.*

2. *Suppose $\alpha^s$ is small enough, then an increase in the extent of seller heterogeneity ($\gamma^s$) increases $\hat{p}^s$, decreases $\hat{p}^b$, and increases $\hat{p}^b + \hat{p}^s$.*

We first discuss the first result ($\gamma^b$) of Proposition 6. We first observe from (18) that the effect of an increase in $\gamma^b$ critically depends on the sign of $\hat{p}^b - \alpha^b$. Suppose $\alpha^b$ is small enough so that $\hat{p}^b - \alpha^b > 0$ (so platform-mediated transactions are relatively costly). An increase in $\gamma^b$ raises the attractiveness of platform-mediated transactions (relative to the outside option) by dampening buyers' sensitivity towards the net cost of using platforms. Buyers are less likely to stop transacting, so that sellers find it easier to divert buyers' transactions, in the sense that index $\sigma$ decreases. This weakens platforms' market power over sellers. As for the buyer side, $\gamma^b$ has standard two effects: (i) increased differentiation between the platforms; and (ii) dampened buyer sensitivity towards the net cost of using platforms, which expands the market size. Both effects raise the market power platforms have over buyers.

The case of $\hat{p}^b - \alpha^b \leq 0$ (so the transaction is subsidized on net) is more complicated.[29] In this case, an increase in $\gamma^b$ decreases the attractiveness of platform-mediated transactions by dampening buyer sensitivity towards the net subsidy of using platforms $\hat{p}^b - \alpha^b \leq 0$. This is in contrast to the previous case of small $\alpha^b$. Consequently, the index $\sigma$ increases, strengthening platforms' market power over the sellers. As for the buyer side, the effect of $\gamma^b$ is unclear because it now has a market contraction effect, which offsets (and potentially dominates) the increase in market power from increased differentiation. The overall effect of $\gamma^b$ on the equilibrium fee is generally ambiguous in this case.

Finally, the the effect of $\gamma^s$ in the second part of Proposition 6 follows a largely similar reasoning, in the sense that its effect depends also on the sign of $\hat{p}^s - \alpha^s$, except that the effects through the loyalty index are absent in this case.

# 7 Conclusion

This paper investigated two-sided market pricing by oligopolistic platforms when platforms set transaction fees on both user sides. We provided a framework which allows the underlying economic forces determining platform pricing to be analyzed. The following table summarizes the findings of our various comparative statics exercises.

| An increase in | Change in total fee | Shift in fee structure |
|---|---|---|
| extent of buyer multihoming ($\lambda$) | − | favors sellers |
| number of platforms ($n$) when $\lambda$ is large | − | favors sellers* |
| number of platforms ($n$) when $\lambda$ is small | − | favors buyers |
| value of transactions for buyers ($\alpha^b$) | + | favors sellers |
| value of transactions for sellers ($\alpha^s$) | + | favors buyers |
| buyer heterogeneity when $\alpha^b$ is small | + | favors sellers |
| seller heterogeneity when $\alpha^s$ is small | + | favors buyers |

Note: " + " = increase; " − " = decrease;

* = with additional conditions specified in Proposition 2.

Our findings echo the general view in the two-sided market literature that one-sided logic may be misleading in making inferences in two-sided markets (e.g., Wright, 2004). First, seemingly buyer-friendly measures that facilitate multihoming and switching across platforms, such as the growing popularity of metasearch aggregators, may primarily benefit sellers by making it more attractive for sellers to divert transactions onto a cheaper platform. Second, in a two-sided context, more platform competition does not necessarily drive down both prices; indeed we find, more platform competition can increase buyer-side prices when there is a lot of multihoming among buyers, and it can increase seller-side prices when there is little multihoming among buyers.

[29] See Anderson et al. (1992) for a similar discussion in the context of oligopolistic firms selling differentiated products.

There are two built-in asymmetries across the two sides in our model (i) buyers choose the platform to make their transaction on; (ii) sellers treat platforms as homogenous. It would be interesting to explore relaxing these. One possibility is to allow sellers to directly influence buyer decisions on which platform to transact on (e.g., allow ride-hailing drivers to observe destinations and reject orders). Another direction is to extend the framework to study what happens if platforms are differentiated from the sellers' perspective as well. This would potentially generate richer equilibrium configurations where some sellers multihome on a subset of platforms while other sellers multihome on all platforms.

It would be interesting to allow platforms to influence homing behaviors of buyer and sellers through strategies such as exclusive contracts, product design, offering subscription contracts, usage bundling (Sato, 2021), or limiting the effectiveness of multihoming tools (Athey et al., 2018). Our current setup is not well suited for this investigation as we rely on symmetry to make our analysis tractable. For a recent advancement in this direction, see Haan et al. (2021).

Finally, one can try to incorporate seller competition more explicitly into the current framework, which is particularly relevant when the sellers are merchants that compete on price. In our analysis of seller participation, an implicit assumption is that potential transactions with each seller are irreplaceable. Allowing for seller competition should increase the buyer loyalty index because it weakens the ability of each seller to divert buyers by delisting from a more expensive platform. Future research could formalize this result, and see whether increased seller competition indeed leads platforms to increase their fees to sellers and in total, while decreasing their fees to buyers.

# A Appendix

## A.1 Further details of demand derivation

In this appendix, we complete the demand derivation by considering the seller participation profile under a downward deviation $p_i^s < \hat{p}^s$. It is obvious that if a seller joins at least one platform, then the seller must also join platform $i$ given that $i$ charges the lowest seller fee. A seller will join $i$ as long as $v \geq p_i^s$. However, the fact that all other platforms $j \neq i$ set $\hat{p}^s$ does not necessarily imply that the seller will join all these platforms together in a "block". This is because when $p_i^s < \hat{p}^s$, any additional platform that a seller joins will divert additional buyers away from the lowest-fee platform $i$ to the newly joined platform. Therefore, the number of platforms a seller multihomes on will depend on $v$ in general.

Consider a seller who chooses to join platform $i$ together with $m-1$ other (symmetric) platforms. We denote this set of platforms as $\mathbf{N}_{i,m}$ (the seller joins $m$ platforms in total, including $i$). Note that $\mathbf{N}_{i,1} = \{i\}$ and $\mathbf{N}_{i,n} = \mathbf{N}$ so $m$ is bounded between 1 and $n$. The corresponding number of buyers who use $i$ for transactions is

$$B_i^{(\mathbf{N}_{i,m})} = \Pr\left(\epsilon_i - p_i^b \geq \max_{j \in \mathbf{N}_{i,m}} \left\{\epsilon_j - p_j^b, \epsilon_0\right\}\right).$$

Clearly a higher $m$ implies more buyers diverted from platform $i$ since $B_i^{(\mathbf{N}_{i,m})}$ decreases with $m$. With a slight abuse of notation, let

$$B_0^{(\mathbf{N}_{i,m})} = \Pr\left(\epsilon_0 \geq \max_{j \in \mathbf{N}_{i,m}} \left\{\epsilon_j - p_j^b, \epsilon_i - p_i^b\right\}\right).$$

The following lemma states sellers' multihoming decision formally:

**Lemma A.1** *Suppose $p_i^s < \hat{p}^s$. For $m = 2, ..., n$, define cutoffs*

$$\hat{v}_m \equiv (\hat{p}^s - p_i^s) \frac{B_i^{(\mathbf{N}_{i,m})} - B_i^{(\mathbf{N}_{i,m+1})}}{B_0^{(\mathbf{N}_{i,m})} - B_0^{(\mathbf{N}_{i,m+1})}} + \hat{p}^s. \tag{19}$$

*A type $v$ seller joins no platform if $v \in [\underline{v}, p_i^s)$, joins only platform $i$ if $v \in [p_i^s, \hat{v}_2)$, joins platform $i$ together with $m-1$ randomly chosen symmetric platform(s) from $j \neq i$ if $v \in [\hat{v}_m, \hat{v}_{m+1})$, and joins all platforms if $v > \hat{v}_n$.*

**Proof.** Consider a type $v$ seller that has joined platform $i$ and that is contemplating whether to join one of the platforms $j \neq i$ in addition. The utility of joining $i$ alone (so $m = 1$) is $(v - p_i^s) B_i^{(\mathbf{N}_{i,0})}$, so this is superior than joining no platforms as long as $v \geq p_i^s$. Meanwhile the utility from joining another platform $j \neq i$ (so that $m = 2$) is $(v - p_i^s) B_i^{(\mathbf{N}_{i,1})} + (v - \hat{p}^s) B_j^{(\mathbf{N}_{i,1})}$. Comparing the two utilities yields the first cutoff

$$\hat{v}_2 \equiv (\hat{p}^s - p_i^s) \frac{B_i^{(\mathbf{N}_{i,0})} - B_i^{(\mathbf{N}_{i,1})}}{-B_i^{(\mathbf{N}_{i,0})} + B_i^{(\mathbf{N}_{i,1})} + B_j^{(\mathbf{N}_{i,1})}} + \hat{p}^s = (\hat{p}^s - p_i^s) \frac{B_i^{(\mathbf{N}_{i,0})} - B_i^{(\mathbf{N}_{i,1})}}{B_0^{(\mathbf{N}_{i,0})} - B_0^{(\mathbf{N}_{i,1})}} + \hat{p}^s.$$

Now suppose a seller has joined the set of platforms $\mathbf{N}_{i,m-1}$, i.e., platform $i$ plus $m-2$ other platforms. Owing to the symmetry of all platforms $j \neq i$, the seller's utility can be written as $(v - p_i^s) B_i^{(\mathbf{N}_{i,m-1})} + (m-2)(v - \hat{p}^s) B_j^{(\mathbf{N}_{i,m-1})}$. The utility of joining one more platform — so that the seller joins platform $i$ plus $m$ other platforms, i.e. the set of platforms $\mathbf{N}_{i,m}$, is $(v - p_i^s) B_i^{(\mathbf{N}_{i,m})} + (m-1)(v - \hat{p}^s) B_j^{(\mathbf{N}_{i,m})}$. Comparing the two utilities yields cutoffs $\hat{v}_m$ (19) for all $m \leq n$. ■

Combining this with the case of upward deviation derived in the main text, the complete demand function faced by platform $i$ is piece-wise defined by

$$Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) = \left\{ \begin{array}{cc} \sum_{m=1}^{n} \left[G(\hat{v}_{m+1}) - G(\hat{v}_m)\right] B_i^{(\mathbf{N}_{i,m})} & \text{if } p_i^s < \hat{p}^s \\ (1 - G(\hat{v})) B_i^{(\mathbf{N})} & \text{if } p_i^s \geq \hat{p}^s \end{array} \right\}, \tag{20}$$

where we denote $\hat{v}_1 \equiv p_i^s$ and $\hat{v}_{n+1} \equiv \bar{v}$ (so that $G(\hat{v}_n) = 1$). Note that when $p_i^s < \hat{p}^s$, the volume takes into account sellers' heterogenous multihoming behavior. Figure 6 provides an illustration of function (20) assuming $n = 3$:

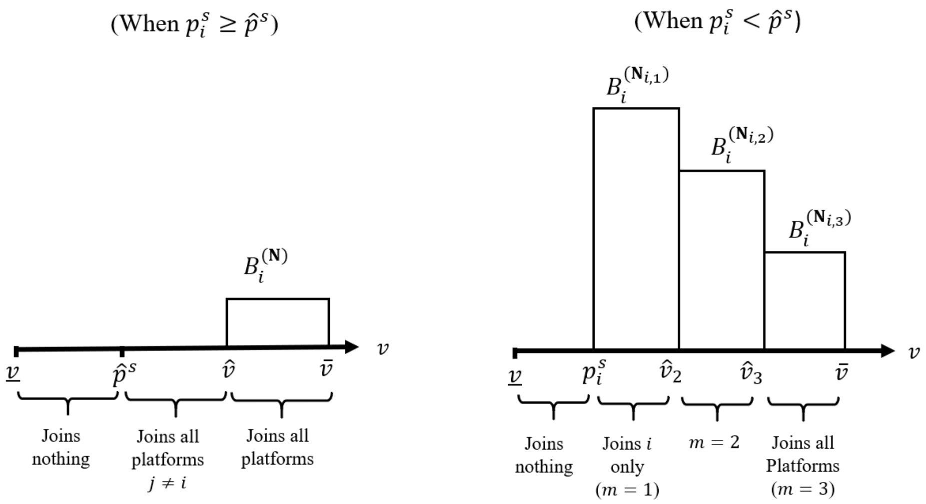


Figure 6: Seller multihoming and the associated transactions by buyers through $i$.

The left panel of Figure 6 depicts $Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right)$ when $p_i^s \geq \hat{p}^s$. In this case, only sellers with $v \geq \hat{v}$ join platform $i$, and the mass of buyers who use platform $i$ to transact with each of these sellers is $B_i^{(\mathbf{N})}$, that is, those who find $i$ most attractive when all $n$ platforms are available for transactions. The right panel of Figure 6 depicts the case of $p_i^s < \hat{p}^s$, where recall $m$ denotes the number of platforms that a seller multihomes on in addition to platform $i$. Sellers with $v \in [p_i^s, \hat{v}_1)$ join platform $i$ exclusively, so that buyers who transact with these sellers can only choose between transacting through $i$ or transacting directly. The mass of buyers who use $i$ to transact with these sellers is $B_i^{(\mathbf{N}_{i,1})}$, that is, those who find $i$ more attractive than the outside option. Sellers with $v \in [\hat{v}_2, \hat{v}_3)$ join platform $i$ and a randomly selected platform $j \neq i$, so that buyers who transact with these sellers can choose between transacting through $i$, $j$, or transacting directly. Notably, the mass of buyers who use $i$ to transact with these sellers is $B_i^{(\mathbf{N}_{i,2})}$, which is smaller than $B_i^{(\mathbf{N}_{i,1})}$ due to the availability of an additional alternative platform for transactions.

If $B_i^{(\cdot)}$ satisfies the IIA property, it implies the ratio $B_i^{(\mathbf{N}_{i,m})}/B_0^{(\mathbf{N}_{i,m})}$ is independent of $m$. Denote the said constant ratio as $\chi$. Through algebraic manipulations, we can simplify $\hat{v}_m$ in (19) as

$$\hat{v}_m = (\hat{p}^s - p_i^s)\,\chi + \hat{p}^s, \tag{21}$$

which is independent of $m$.

## A.2 Proof of Proposition 1

We first state and prove the following two lemmas, which also prove Lemma 2 in the main text.

**Lemma A.2** *Buyer-side inverse semi-elasticity $X(p;n)$ defined in (6) is decreasing in $n$ and $p$.*

**Proof.** Let $\epsilon_{(n)}$ denote the highest order statistic (out of $n$ draws of $\epsilon$), and denote

$$\bar{X}(\epsilon_0 + p; n) = \frac{\frac{1}{n}\left(1 - F(\epsilon_0 + p)^n\right)}{\int_{\epsilon_0+p}^{\bar{\epsilon}} [f(\epsilon)]\, dF(\epsilon)^{n-1} + f(\epsilon_0 + p)\, F(\epsilon_0 + p)^{n-1}}$$

as the buyer inverse semi-elasticity for given non-random outside option $\epsilon_0 + p$. Then, from definition (6) and exploiting the alternative expression of

$$\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \int_{\underline{\epsilon}}^{\bar{\epsilon}} 1 - F\left(\max\left\{\epsilon, \epsilon_0 + p^b\right\}\right) dF(\epsilon)^{n-1}\, dF_0(\epsilon_0) = \frac{1}{n} \int_{\underline{\epsilon}}^{\bar{\epsilon}-p} \left[1 - F(\epsilon_0 + p)^n\right] dF_0(\epsilon_0),$$

we can rewrite $X(p;n)$ as

$$\begin{aligned}
\frac{1}{X(p;n)} &= \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \left[\frac{\int_{\epsilon_0+p}^{\bar{\epsilon}} [f(\epsilon)]\, dF(\epsilon)^{n-1} + f(\epsilon_0+p)\, F(\epsilon_0+p)^{n-1}}{\frac{1}{n}\left(1 - F(\epsilon_0+p)^n\right)}\right] \left[\frac{\left(1 - F(\epsilon_0+p)^n\right)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}-p} \left[1 - F(\epsilon_0+p)^n\right] dF_0(\epsilon_0)}\right] dF_0(\epsilon_0) \\
&= \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \left[\frac{1}{\bar{X}(\epsilon_0+p;n)}\right] \left[\frac{\left(1 - F(\epsilon_0+p)^n\right)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}-p} \left[1 - F(\epsilon_0+p)^n\right] dF_0(\epsilon_0)}\right] dF_0(\epsilon_0).
\end{aligned}$$

Define a new random variable $\tilde{\epsilon}_0 \equiv \epsilon_0 + p$ with support over $[\underline{\epsilon}_0 + p, \bar{\epsilon}_0 + p]$, and define the cdf of $\tilde{\epsilon}_0$ conditioned on it being smaller than $\epsilon_{(n)}$:

$$H(x;n,p) \equiv \Pr\left(\tilde{\epsilon}_0 < x | \tilde{\epsilon}_0 < \epsilon_{(n)}\right) = \frac{\int_{\underline{\epsilon}_0+p}^{x} \left(1 - F(\tilde{\epsilon}_0)^n\right) f_0(\tilde{\epsilon}_0 - p)\, d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}} \left(1 - F(\tilde{\epsilon}_0)^n\right) f_0(\tilde{\epsilon}_0 - p)\, d\tilde{\epsilon}_0}. \tag{22}$$

Then,

$$\frac{1}{X(p;n)} = \int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p} \left[\frac{1}{\bar{X}(\tilde{\epsilon}_0;n)}\right] dH(\tilde{\epsilon}_0;n,p).$$

Lemma 4 of Zhou (2017) shows that $1/\bar{X}(\tilde{\epsilon}_0;n)$ is increasing in $\tilde{\epsilon}_0$ and $n$. Hence, to conclude that $\frac{1}{X(p;n)}$ is increasing in $p$ and $n$, it remains to show that the conditional random variable $\tilde{\epsilon}_0|_{\tilde{\epsilon}_0 < \epsilon_{(n)}}$ is increasing in $n$ and $p$ in the sense of first-order stochastic dominance (FOSD), i.e. $H(x;n,p)$ is decreasing in $p$ and $n$ at each given $x$.

**Claim:** $\tilde{\epsilon}_0|_{\tilde{\epsilon}_0 < \epsilon_{(n)}}$ *is FOSD increasing in* $p$. From the cdf function, the relevant derivative $\frac{\partial H(x;n,p)}{\partial p}$ can be shown to be negative if

$$\frac{\int_{\underline{\epsilon}_0+p}^{x} \left[1 - F(\tilde{\epsilon}_0)^n\right] f_0'(\tilde{\epsilon}_0 - p)\, d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{x} \left[1 - F(\tilde{\epsilon}_0)^n\right] f_0(\tilde{\epsilon}_0 - p)\, d\tilde{\epsilon}_0} \geq \frac{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p} \left[1 - F(\tilde{\epsilon}_0)^n\right] f_0'(\tilde{\epsilon}_0 - p)\, d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p} \left[1 - F(\tilde{\epsilon}_0)^n\right] f_0(\tilde{\epsilon}_0 - p)\, d\tilde{\epsilon}_0}. \tag{23}$$

Given $x \leq \bar{\epsilon}$, establishing (23) is equivalent to showing that the left-hand side of (23) is decreasing in $x$. If we define the distribution function

$$\begin{aligned}\tilde{H}(y;x) &= \Pr\left(\tilde{\epsilon}_0 < y|\tilde{\epsilon}_0 < \max\left\{\epsilon_{(n)}, x\right\}\right) \\ &= \frac{\int_{\underline{\epsilon}_0+p}^{y}\left[1-F(\tilde{\epsilon}_0)^n\right] f_0(\tilde{\epsilon}_0-p)\, d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{x}\left[1-F(\tilde{\epsilon}_0)^n\right] f_0(\tilde{\epsilon}_0-p)\, d\tilde{\epsilon}_0} \quad \text{for } y \in [\underline{\epsilon}+p, x],\end{aligned}$$

then we can rewrite the left-hand side of (23) as $\int_{\underline{\epsilon}_0+p}^{x}\left[\frac{f_0'(y-p)}{f_0(y-p)}\right] d\tilde{H}(y;x)$. Log-concavity of $f_0$ implies that $\frac{f_0'(y-p)}{f_0(y-p)}$ is decreasing in $y$. Meanwhile it is easily verified from its definition, that $\tilde{H}(y;x)$ is FOSD increasing in $x$. Therefore, we conclude that the left-hand side of (23) is decreasing in $x$, so that inequality (23) indeed holds.

**Claim:** *$\tilde{\epsilon}_0|_{\tilde{\epsilon}_0<\epsilon_{(n)}}$ is FOSD increasing in $n$.* From the cdf function, the relevant derivative $\frac{\partial H(x;n,p)}{\partial n}$ can be shown to be negative if

$$\frac{\int_{\underline{\epsilon}_0+p}^{x}\left[-\ln F(\tilde{\epsilon}_0) F(\tilde{\epsilon}_0)^n\right] f_0(\tilde{\epsilon}_0-p)\, d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{x}\left[1-F(\tilde{\epsilon}_0)^n\right] f_0(\tilde{\epsilon}_0-p)\, d\tilde{\epsilon}_0} \leq \frac{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\left[-\ln F(\tilde{\epsilon}_0) F(\tilde{\epsilon}_0)^n\right] f_0(\tilde{\epsilon}_0-p)\, d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\left[1-F(\tilde{\epsilon}_0)^n\right] f_0(\tilde{\epsilon}_0-p)\, d\tilde{\epsilon}_0}, \tag{24}$$

so that $\frac{\partial H(x;n,p)}{\partial n} \leq 0$ if the left-hand side of (24) is increasing in $x$. Applying the same technique used in the previous claim, we can write the left-hand side of (24) as

$$\int_{\underline{\epsilon}_0+p}^{x}\left[\frac{-\ln F(y) F(y)^n}{1-F(y)^n}\right] d\tilde{H}(y;x).$$

Since $-\ln F(y) \geq 0$, we know that $\frac{-\ln F(y) F(y)^n}{1-F(y)^n}$ is increasing in $y$. This fact, together with the fact that $\tilde{H}(y;x)$ is FOSD increasing in $x$, implies that the left-hand side of (24) is increasing in $x$, and so the inequality in (24) indeed holds. ■

**Lemma A.3** *The buyer loyalty index $\sigma(p;n)$ defined in (7) is strictly decreasing in $n$ and increasing in $p$.*

**Proof.** Recall $\sigma(p;n)$ is

$$\sigma(p;n) = \frac{\Pr\left(\epsilon_0 \geq \max_{j\in\mathbf{N}_{-i}}\{\epsilon_j - p\}\right) - \Pr\left(\epsilon_0 \geq \max_{j\in\mathbf{N}}\{\epsilon_j - p\}\right)}{\Pr\left(\epsilon_i \geq \max_{j\in\mathbf{N}}\{\epsilon_j - p, \epsilon_0\}\right)}.$$

Rewrite it as:

$$\begin{aligned}\sigma(p;n) &= \frac{\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\left[nF(\epsilon_0+p)^{n-1}(1-F(\epsilon_0+p))\right] dF_0(\epsilon_0)}{\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\left[1-F(\epsilon_0+p)^n\right] dF_0(\epsilon_0)} \\ &= \Pr\left(\epsilon_{(n-1)} < \tilde{\epsilon}_0|\epsilon_{(n)} > \tilde{\epsilon}_0\right),\end{aligned}$$

where $\tilde{\epsilon}_0 \equiv \epsilon_0 + p$. To show $\sigma(p;n)$ strictly increases with $p$, we write

$$\begin{aligned}\sigma(p;n) &= \int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p} \Pr\left(\epsilon_{(n-1)} < y|\epsilon_{(n)} > y\right) \Pr\left(\tilde{\epsilon}_0 = y|\tilde{\epsilon}_0 < \epsilon_{(n)}\right) dy \\ &= \int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p} \Pr\left(\epsilon_{(n-1)} < y|\epsilon_{(n)} > y\right) dH(\tilde{\epsilon}_0;n,p),\end{aligned} \tag{25}$$

where $H(\tilde{\epsilon}_0;n,p)$ is the conditional distribution function defined in (22). We first observe that $\Pr\left(\epsilon_{(n-1)} < y|\epsilon_{(n)} > y\right)$ is strictly increasing in $y$:

$$\begin{aligned}\Pr\left(\epsilon_{(n-1)} < y|\epsilon_{(n)} > y\right) &= \frac{nF(y)^{n-1}(1-F(y))}{(1-F(y)^n)} \\ \frac{d}{dy}\Pr\left(\epsilon_{(n-1)} < y|\epsilon_{(n)} > y\right) &= \frac{1-F(y)-\frac{1}{n}(1-F(y)^n)}{(1-F(y)^n)^2} f(y)\, n^2 > 0.\end{aligned}$$

We also know from the proof of Lemma A.2 that the conditional random variable $\tilde{\epsilon}_0|_{\tilde{\epsilon}_0<\epsilon_{(n)}}$ associated with cdf $H$ is FOSD increasing in $p$. This fact, together with the observation that the integrand of (25) is an increasing function, imply that $\sigma(p;n)$ is strictly increasing in $p$ as required.

To show $\sigma(p;n)$ strictly decreases with $n$, we write

$$\sigma(p;n) = \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \Pr\left(\epsilon_{(n-1)} < \tilde{\epsilon}_0 | \epsilon_{(n)} = y\right) \Pr\left(\epsilon_{(n)} = y | \epsilon_{(n)} > \tilde{\epsilon}_0\right) dy.$$

Then, we make the following two claims:

**Claim:** *For arbitrary constant $y \in [\underline{\epsilon}, \bar{\epsilon}]$, $\Pr\left(\epsilon_{(n-1)} < \tilde{\epsilon}_0 | \epsilon_{(n)} = y\right)$ is strictly decreasing in $n$ and $y$.* By definition,

$$\begin{aligned}\Pr\left(\epsilon_{(n-1)} < \tilde{\epsilon}_0 | \epsilon_{(n)} = y\right) &= \frac{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p} nF\left(\min\left\{\tilde{\epsilon}_0, y\right\}\right)^{n-1} f(y)\, dF_0(\tilde{\epsilon}_0 - p)}{nF(y)^{n-1} f(y)} \\ &= \int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p} \left(\frac{F\left(\min\left\{\tilde{\epsilon}_0, y\right\}\right)}{F(y)}\right)^{n-1} dF_0(\tilde{\epsilon}_0 - p),\end{aligned}$$

which is clearly strictly decreasing in $n$ and $y$.

**Claim:** *$\epsilon_{(n)}|_{\epsilon_{(n)}>\tilde{\epsilon}_0}$ is FOSD increasing in $n$.* By definition, the corresponding CDF is

$$\begin{aligned}\Pr\left(\epsilon_{(n)} < x | \epsilon_{(n)} > \tilde{\epsilon}_0\right) &= \frac{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p} \left[F(x)^n - F\left(\min\left\{\tilde{\epsilon}_0, x\right\}\right)^n\right] dF_0(\tilde{\epsilon}_0 - p)}{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p} \left[1 - F\left(\min\left\{\tilde{\epsilon}_0, x\right\}\right)^n\right] dF_0(\tilde{\epsilon}_0 - p)} \\ &= \int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p} \left[\frac{F(x)^n - F\left(\min\left\{\tilde{\epsilon}_0, x\right\}\right)^n}{1 - F\left(\min\left\{\tilde{\epsilon}_0, x\right\}\right)^n}\right] dH(\tilde{\epsilon}_0; n, p),\end{aligned}$$

where $H(\tilde{\epsilon}_0; n, p)$ is the conditional distribution function defined in (22). We first observe that the integrand is decreasing in $n$: showing this is equivalent to showing that an arbitrary sequence $\frac{a^n-1}{b^n-1}$, $(1 < a < b)$ is decreasing in $n$, which is easily verified. Likewise, the integrand is decreasing in $\tilde{\epsilon}_0$. These two observations, together with the proven result that the conditional random variable $\tilde{\epsilon}_0|_{\tilde{\epsilon}_0<\epsilon_{(n)}}$ associated with cdf $H$ is FOSD increasing in $n$, implies $\Pr\left(\epsilon_{(n)} < x | \epsilon_{(n)} > \tilde{\epsilon}_0\right)$ is decreasing in $n$ as required.

Using these two claims, we have for any $n' \geq n$,

$$\begin{aligned}& \sigma(p;n) \\ > & \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \Pr\left(\epsilon_{(n'-1)} < \tilde{\epsilon}_0 | \epsilon_{(n')} = y\right) \Pr\left(\epsilon_{(n)} = y | \epsilon_{(n)} > \tilde{\epsilon}_0\right) dy \\ \geq & \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \Pr\left(\epsilon_{(n'-1)} < \tilde{\epsilon}_0 | \epsilon_{(n')} = y\right) \Pr\left(\epsilon_{(n')} = y | \epsilon_{(n')} > \tilde{\epsilon}_0\right) dy \\ = & \Pr\left(\epsilon_{(n'-1)} < \tilde{\epsilon}_0 | \epsilon_{(n')} > \tilde{\epsilon}_0\right) = \sigma(p; n').\end{aligned}$$

So $\sigma(p;n)$ is indeed strictly decreasing in $n$. ■

To prove Proposition 1, we first note that the demand derivatives, after imposing symmetry, are

$$\begin{aligned}Q_i(\hat{\mathbf{p}}; \hat{\mathbf{p}}) &= \left(1 - G(\hat{p}^s)\right) B_i^{(\mathbf{N})}|_{p_i^b=\hat{p}^b} \\ &= \left(1 - G(\hat{p}^s)\right) \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \int_{\underline{\epsilon}}^{\bar{\epsilon}} 1 - F\left(\max\left\{\epsilon, \epsilon_0 + \hat{p}^b\right\}\right) dF(\epsilon)^{n-1} dF_0(\epsilon_0) \\ \frac{dQ_i(\hat{\mathbf{p}}; \hat{\mathbf{p}})}{dp_i^b} &= \left(1 - G(\hat{p}^s)\right) \frac{\partial B_i^{(\mathbf{N})}}{\partial p_i^b}|_{p_i^b=\hat{p}^b} \\ &= -\left(1 - G(\hat{p}^s)\right) \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f\left(\max\left\{\epsilon, \epsilon_0 + \hat{p}^b\right\}\right) dF(\epsilon)^{n-1} dF_0(\epsilon_0). \\ \frac{dQ_i(\hat{\mathbf{p}}; \hat{\mathbf{p}})}{dp_i^s} &= -g(\hat{p}^s) \frac{d\hat{v}}{dp_i^s} B_i^{(\mathbf{N})}|_{p_i^b=\hat{p}^b} \\ &= \frac{-g(\hat{p}^s)}{\sigma(p^b; n)} B_i^{(\mathbf{N})}|_{p_i^b=\hat{p}^b}.\end{aligned}$$

The standard first-order condition yields (8). To prove the existence and uniqueness of $\hat{p}^b$ and $\hat{p}^s$, we rearrange the first equality in (8) as $\hat{p}^s = c + X(\hat{p}^b) - \hat{p}^b$ and substitute it into the second equality to get

$$\frac{X(\hat{p}^b)}{\sigma(\hat{p}^b)} = \frac{1 - G(c + X(\hat{p}^b) - \hat{p}^b)}{g(c + X(\hat{p}^b) - \hat{p}^b)}. \tag{26}$$

The left-hand side of (26) is decreasing in $\hat{p}^b$ by Lemmas A.2 and A.3, while the right-hand side of (26) is increasing in $\hat{p}^b$ by log-concavity of $1 - G$ and Lemma A.2. Hence, any solution $\hat{p}^b$ to (26) must be unique, while $\hat{p}^s$ can be uniquely determined from the first equality in (8).

## A.3 Proof of Proposition 2

Denote $M(\hat{p}^s) \equiv \frac{1-G(\hat{p}^s)}{g(\hat{p}^s)}$, and let the derivatives of $X$ and $\sigma$ with respect to the first argument be denoted $X'$ and $\sigma'$. Total differentiation of (8), in matrix form, gives

$$\begin{bmatrix} 1 - X' & 1 \\ 1 - M\sigma' & 1 - \sigma\frac{\partial M}{\partial \hat{p}^s} \end{bmatrix} \begin{bmatrix} \frac{d\hat{p}^b}{dn} \\ \frac{d\hat{p}^s}{dn} \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial n} \\ M\frac{\partial \sigma}{\partial n} \end{bmatrix}. \tag{27}$$

Since $X' \leq 0$, $\frac{\partial M}{\partial \hat{p}^s} < 0$, and $\sigma' > 0$ (Lemma A.2 and Lemma A.3), the matrix in (27) has determinant

$$Det \equiv \underbrace{\left(1 - X'\right)\left(1 - \sigma\frac{\partial M}{\partial \hat{p}^s}\right)}_{>1} - 1 + M\sigma' > 0. \tag{28}$$

By Cramer's rule, and substituting for the equilibrium condition $M = \frac{X}{\sigma}$, we have

$$\frac{d\hat{p}^s}{dn} = \frac{1}{Det}\begin{vmatrix} 1 - X' & \frac{\partial X}{\partial n} \\ 1 - M\sigma' & M\frac{\partial \sigma}{\partial n} \end{vmatrix} = \frac{X}{Det}\left[\underbrace{-\frac{\partial \sigma}{\partial n}\frac{X'}{\sigma}}_{\leq 0} + \underbrace{\frac{\partial X}{\partial n}\frac{\sigma'}{\sigma}}_{\leq 0} + \left(\frac{1}{\sigma}\frac{\partial \sigma}{\partial n} - \frac{1}{X}\frac{\partial X}{\partial n}\right)\right]; \tag{29}$$

$$\frac{d\hat{p}^b}{dn} = \frac{1}{Det}\begin{vmatrix} \frac{\partial X}{\partial n} & 1 \\ M\frac{\partial \sigma}{\partial n} & 1 - \sigma\frac{\partial M}{\partial \hat{p}^s} \end{vmatrix} = \frac{X}{Det}\left[\underbrace{-\frac{\sigma}{X}\frac{\partial M}{\partial \hat{p}^s}\frac{\partial X}{\partial n}}_{\leq 0} - \left(\frac{1}{\sigma}\frac{\partial \sigma}{\partial n} - \frac{1}{X}\frac{\partial X}{\partial n}\right)\right], \tag{30}$$

where $\frac{1}{\sigma}\frac{\partial \sigma}{\partial n} - \frac{1}{X}\frac{\partial X}{\partial n}$ indicates the net see-saw effect. We know $\frac{\partial X}{\partial n} \leq 0$ and $\frac{\partial \sigma}{\partial n} \leq 0$ (Lemma A.2 and Lemma A.3), therefore

$$\frac{d\hat{p}^s}{dn} + \frac{d\hat{p}^b}{dn} = \frac{1}{Det}\left[M\left(\underbrace{\sigma'\frac{\partial X}{\partial n}}_{<0} - \underbrace{\frac{\partial \sigma}{\partial n}X'}_{>0}\right) - \sigma\underbrace{\frac{\partial M}{\partial \hat{p}^s}\frac{\partial X}{\partial n}}_{\geq 0}\right] \leq 0.$$

Denote

$$\Lambda \equiv \int_{\underline{\epsilon}}^{\bar{\epsilon}-p}\int_0^{\bar{\epsilon}} f\left(\max\left\{\epsilon, \epsilon_0 + \hat{p}^b\right\}\right) dF(\epsilon)^{n-1} dF_0(\epsilon_0),$$

and let $\Lambda'$ be its partial derivative wrt $n$. If $f$ is decreasing, the fact that $F(\epsilon)^{n-1}$ is FOSD increasing in $n$ implies $\Lambda' \leq 0$. Computing the relevant derivatives, we get

$$\begin{aligned} \frac{\partial \sigma/\partial n}{\sigma} - \frac{\partial X/\partial n}{X} &= \left(\frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left[\ln\left(F\left(\epsilon_0 + \hat{p}^b\right)\right) F\left(\epsilon_0 + \hat{p}^b\right)^{n-1}\left(1 - F\left(\epsilon_0 + \hat{p}^b\right)\right)\right] dF_0(\epsilon_0)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left[F(\epsilon_0 + \hat{p}^b)^{n-1}(1 - F(\epsilon_0 + \hat{p}^b))\right] dF_0(\epsilon_0)} + \frac{\Lambda'}{\Lambda}\right) \\ &< 0, \end{aligned}$$

where the inequality is due to $\ln\left(F\left(\epsilon_0 + \hat{p}^b\right)\right) < 0$ and $\Lambda' \leq 0$. Thus, $\frac{d\hat{p}^s}{dn} < 0$ from (29).

If $g$ is decreasing, it implies that $\frac{\partial M}{\partial \hat{p}^s} \geq -1$ so (30) implies

$$\frac{d\hat{p}^b}{dn} \geq \frac{1}{Det}\left[\left((1+\sigma)\frac{1}{X}\frac{\partial X}{\partial n} - \frac{1}{\sigma}\frac{\partial \sigma}{\partial n}\right)\right]. \tag{31}$$

The right-hand side of (31) is strictly positive if and only if

$$
\begin{aligned}
0 \quad > \quad & \frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left[\ln\left(F\left(\epsilon_0+\hat{p}^b\right)\right)F\left(\epsilon_0+\hat{p}^b\right)^{n-1}\left(1-F\left(\epsilon_0+\hat{p}^b\right)\right)\right]dF_0\left(\epsilon_0\right)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}-p}\left[F\left(\epsilon_0+\hat{p}^b\right)^{n-1}\left(1-F\left(\epsilon_0+\hat{p}^b\right)\right)\right]dF_0\left(\epsilon_0\right)} \\
& +\sigma\left(\frac{1}{n}+\frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left[\ln\left(F\left(\epsilon_0+\hat{p}^b\right)\right)F\left(\epsilon_0+\hat{p}^b\right)^{n}\right]dF_0\left(\epsilon_0\right)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left[1-F\left(\epsilon_0+\hat{p}^b\right)^n\right]dF_0\left(\epsilon_0\right)}\right)+\frac{\Lambda'}{\Lambda}\left(1+\sigma\right).
\end{aligned}
$$

We know $\Lambda' \leq 0$ if $f$ is weakly decreasing. Meanwhile, applying L'Hopital rule twice shows that the first two components converges to zero when $\hat{p}^b \to (\bar{\epsilon}-\underline{\epsilon})$. Moreover, calculating the first derivative shows that the sum of the first two components to be increasing in $\hat{p}^b$, hence the sum is non-positive for all $\hat{p}^b \leq \bar{\epsilon}-\underline{\epsilon}$.

## A.4 Proof of Proposition 3

Denote $M \equiv \frac{1-G(\hat{p}^s)}{g(\hat{p}^s)}$. Applying total differentiation with respect to $\lambda$ on (17) and writing in matrix form,

$$
\begin{bmatrix} 1-X' & 1 \\ 1-M\sigma_\lambda' & 1-\sigma_\lambda\frac{\partial M}{\partial \hat{p}^s} \end{bmatrix}\begin{bmatrix} \frac{d\hat{p}^b}{d\lambda} \\ \frac{d\hat{p}^s}{d\lambda} \end{bmatrix}=\begin{bmatrix} 0 \\ M\frac{\partial \sigma_\lambda}{\partial\lambda} \end{bmatrix}.
$$

Using the decomposition of $\sigma_\lambda = \lambda\sigma + 1 - \lambda$, where $\sigma \in [0,1]$ is defined in (7), we have

$$
\begin{bmatrix} 1-X' & 1 \\ 1-\lambda M\sigma' & 1-(\lambda\sigma+1-\lambda)\frac{\partial M}{\partial \hat{p}^s} \end{bmatrix}\begin{bmatrix} \frac{d\hat{p}^b}{d\lambda} \\ \frac{d\hat{p}^s}{d\lambda} \end{bmatrix}=\begin{bmatrix} 0 \\ -M\left(1-\sigma\right) \end{bmatrix},
$$

where

$$
\overline{Det} \equiv \begin{vmatrix} 1-X' & 1 \\ 1-\lambda M\sigma' & 1-(\lambda\sigma+1-\lambda)\frac{\partial M}{\partial \hat{p}^s} \end{vmatrix} = \underbrace{\left(1-X'\right)\left(1-(\lambda\sigma+1-\lambda)\frac{\partial M}{\partial \hat{p}^s}\right)}_{\geq 1}-1+\underbrace{\lambda M\sigma'}_{>0}>0. \tag{32}
$$

By Cramer's rule,

$$
\frac{d\hat{p}^b}{d\lambda}=\frac{M\left(1-\sigma\right)}{\overline{Det}}>0 \quad \text{and} \quad \frac{d\hat{p}^s}{d\lambda}=\frac{-\left(1-X'\right)M\left(1-\sigma\right)}{\overline{Det}}<0.
$$

## A.5 Proof of Proposition 4

Similar to the proof of Proposition 3, we apply total differentiation with respect to $n$ on (17) and obtain matrix form,

$$
\begin{bmatrix} 1-X' & 1 \\ 1-\lambda M\sigma' & 1-(\lambda\sigma+1-\lambda)\frac{\partial M}{\partial \hat{p}^s} \end{bmatrix}\begin{bmatrix} \frac{d\hat{p}^b}{dn} \\ \frac{d\hat{p}^s}{dn} \end{bmatrix}=\begin{bmatrix} \frac{\partial X}{\partial n} \\ \lambda M\frac{\partial \sigma}{\partial n} \end{bmatrix}.
$$

Let $\overline{Det}$ be scalar (32). By Cramer's rule:

$$
\frac{d\hat{p}^s}{dn}=\frac{1}{\overline{Det}}\begin{vmatrix} 1-X' & \frac{\partial X}{\partial n} \\ 1-\lambda M\sigma' & \lambda M\frac{\partial\sigma}{\partial n} \end{vmatrix}.
$$

We next decompose $\frac{d\hat{p}^s}{dn}$ into two components that corresponds to the derivative with respect to $n$ when $\lambda \to 1$ and when $\lambda \to 0$:

$$
\begin{aligned}
\frac{d\hat{p}^s}{dn} &= \frac{1}{\overline{Det}}\left(\lambda\left(1-X'\right)M\frac{\partial\sigma}{\partial n}-\lambda\left(1-M\sigma'\right)\frac{\partial X}{\partial n}+\left(1-\lambda\right)\frac{\partial X}{\partial n}\right) \\
&= \frac{1}{\overline{Det}}\left(Det\times\left(\lambda\frac{d\hat{p}^s}{dn}|_{\lambda=1}\right)-\left(1-\lambda\right)\frac{\partial X}{\partial n}\right),
\end{aligned}
$$

where $Det$ is scalar (28) and $\frac{d\hat{p}^s}{dn}|_{\lambda=1}$ is (29). Following similar steps, we can decompose $\frac{d\hat{p}^b}{dn}$ as

$$
\frac{d\hat{p}^b}{dn}=\frac{1}{\overline{Det}}\left(Det\times\left(\lambda\frac{d\hat{p}^b}{dn}|_{\lambda=1}\right)+\left(1-\lambda\right)\frac{\partial X}{\partial n}\left(1-\frac{\partial M}{\partial \hat{p}^s}\right)\right),
$$

where $\frac{d\hat{p}b}{dn}|_{\lambda=1}$ is (30).

If $\lambda \to 0$, then $\frac{d\hat{p}^s}{dn} \to \frac{-1}{\overline{Det}}\frac{\partial X}{\partial n} \geq 0$ and $\frac{d\hat{p}^b}{dn} \to \frac{1}{\overline{Det}}\frac{\partial X}{\partial n}\left(1-\frac{\partial M}{\partial \hat{p}^s}\right) \leq 0$. Next, if $f$ and $g$ are decreasing then $\frac{d\hat{p}^s}{dn}|_{\lambda=1} < 0$ and $\frac{d\hat{p}^b}{dn}|_{\lambda=1} > 0$ by Proposition 2. Thus, if $\lambda \to 1$, continuity implies $\frac{d\hat{p}^s}{dn} \to \frac{d\hat{p}^s}{dn}|_{\lambda=1} < 0$ and $\frac{d\hat{p}^b}{dn} \to \frac{d\hat{p}^b}{dn}|_{\lambda=1} > 0$. Finally, the sum is

$$\frac{d\hat{p}^s}{dn}+\frac{d\hat{p}^b}{dn} = \lambda\frac{Det}{\overline{Det}}\left(\frac{d\hat{p}^b}{dn}|_{\lambda=1}+\frac{d\hat{p}^s}{dn}|_{\lambda=1}\right)+\frac{1-\lambda}{\overline{Det}}\frac{\partial X}{\partial n}\left(-\frac{\partial M}{\partial \hat{p}^s}\right) \leq 0$$

by Proposition 2.

## A.6 Results for logit buyer quasi-demand

In this section, we analyze the case of $F$ and $F_0 \sim Gumbel(\mu)$ with a common scale parameter $\mu$, where (9) applies. We first prove the quasi-concavity of the profit function when $G$ is linear:

**Lemma A.4** *If $F$, $F_0 \sim Gumbel(\mu)$, and $G$ is linear over $[\underline{v}, \bar{v}]$, then for all $\mathbf{p}_i = (p_i^s, p_i^b)$,*

$$Q_i(\mathbf{p}_i; \hat{\mathbf{p}}) = \left(1-\frac{\hat{v}_m-\underline{v}}{\hat{v}_m-\underline{v}}\right) \times \frac{\exp\{-p_i^b/\mu\}}{1+\exp\{-p_i^b/\mu\}+(n-1)\exp\{-\hat{p}^b/\mu\}},$$

*where $\hat{v}_m = p_i^s + (p_i^s - \hat{p}^s)(n-1)\exp\{-\hat{p}^b/\mu\}$. Moreover, $Q_i(\mathbf{p}_i; \hat{\mathbf{p}})$ is globally log-concave in $\mathbf{p}_i$.*

**Proof.** We first consider $p_i^s \geq \hat{p}^s$. Substituting for the logit demand form and simplifying, Lemma 1 implies the result immediately. When $p_i^s < \hat{p}^s$, logit demand form and (21) implies

$$\hat{v}_m = (\hat{p}^s - p_i^s)\frac{B_i^{(\mathbf{N}_{i,m})}}{B_0^{(\mathbf{N}_{i,m})}} + \hat{p}^s = (\hat{p}^s - p_i^s)\exp\left\{-p_i^b/\mu\right\} + \hat{p}^s,$$

which is independent of $m$, so

$$\begin{aligned}
&Q_i(\mathbf{p}_i; \hat{\mathbf{p}})|_{p_i^s<\hat{p}^s} \\
=& \left(1-G(\hat{v}_m)+(G(\hat{v}_m)-G(p_i^s))\frac{B_i^{(\mathbf{N}_{i,1})}}{B_i^{(\mathbf{N}_{i,n})}}\right)B_i^{(\mathbf{N}_{i,n})} \\
=& \left(1-G(\hat{v}_m)+(G(\hat{v}_m)-G(p_i^s))\left(1+\frac{(n-1)\exp\{-\hat{p}^b/\mu\}}{1+\exp\{-p_i^b/\mu\}}\right)\right)B_i^{(\mathbf{N}_{i,n})} \\
=& \left(1-G(p_i^s)+\left(\frac{G(\hat{v}_m)-G(p_i^s)}{\hat{v}_m-p_i^s}\right)(\hat{p}^s-p_i^s)(n-1)\exp\left\{-\hat{p}^b/\mu\right\}\right)B_i^{(\mathbf{N})}
\end{aligned}$$

where the final line uses $\hat{v}_m - p_i^s = (1+\exp\{-p_i^b/\gamma\})(\hat{p}^s - p_i^s)$ and $B_i^{(\mathbf{N}_{i,n-1})} = B_i^{(\mathbf{N})}$. Linearity of $G$ implies

$$\begin{aligned}
Q_i(\mathbf{p}_i; \hat{\mathbf{p}})|_{p_i^s<\hat{p}^s} &= \left(1-\left(\frac{p_i^s-\underline{v}}{\bar{v}-\underline{v}}\right)+\left(\frac{1}{\bar{v}-\underline{v}}\right)(\hat{p}^s-p_i^s)(n-1)\exp\left\{-\hat{p}^b/\mu\right\}\right)B_i^{(\mathbf{N})} \\
&= \left(1-\frac{\hat{v}_m-\underline{v}}{\bar{v}-\underline{v}}\right)B_i^{(\mathbf{N})}.
\end{aligned}$$

Finally, $Q_i(\mathbf{p}_i; \hat{\mathbf{p}})$ is multiplicatively separable in $p_i^s$ and $p_i^b$, whereby each multiplicative component is obviously log-concave in $p_i^s$ and $p_i^b$ respectively (a logit-demand form is necessarily log-concave). Given that log-concavity is preserved by multiplication, we conclude that $Q_i(\mathbf{p}_i; \hat{\mathbf{p}})$ is log-concave in $\mathbf{p}_i = (p_i^s, p_i^b)$. ■

The following lemma is analogous to the second part of Proposition 2 in the main text. Note that the condition (33) below requires that seller quasi-demand $1-G$ is not too log-concave, that is, $\epsilon_s$ is not too high relative to $n$. We first note that $\epsilon_s > 0$ if $1-G$ is strictly log-concave, $\epsilon_s \geq 1$ if $1-G$ is concave, and $\epsilon_s \leq 1$ if $1-G$ is convex. The latter implies that (33) is immediately satisfied by all distributions with weakly decreasing densities (whereby $1-G$ is convex).

**Lemma A.5** *Suppose $F$, $F_0 \sim Gumbel(\mu)$. In the equilibrium characterized by Proposition 1, an increase in $n$*

*always decreases seller fee* $\hat{p}^s$*, and increases buyer fee* $\hat{p}^b$ *if in addition*

$$4(n-1) > \epsilon_s(p) \ \text{for all } p \in [\underline{v}, \bar{v}], \tag{33}$$

*where* $\epsilon_s(p) \equiv -\frac{d}{dp}\left(\frac{1-G(p)}{g(p)}\right)$ *is the log-curvature index of seller quasi-demand.*

**Proof.** Following the proof of Proposition 2, it suffices to verify

$$\frac{1}{\sigma}\frac{\partial \sigma}{\partial n} - \frac{1}{X}\frac{\partial X}{\partial n} = \frac{-\exp\left\{-\hat{p}^b/\mu\right\}}{1+n\exp\left\{-\hat{p}^b/\mu\right\}} < 0,$$

which follows from simple algebraic manipulations. Meanwhile, $\frac{\partial M}{\partial \hat{p}^s} = -\epsilon_s(p) \leq 0$ by definition, so (30) implies

$$\frac{d\hat{p}^b}{dn} = \frac{1}{Det}\left[\frac{1}{X}\frac{\partial X}{\partial n} - \frac{1}{\sigma}\frac{\partial \sigma}{\partial n} + \epsilon_s \frac{\sigma}{X}\frac{\partial X}{\partial n}\right].$$

Substituting for the corresponding expressions,

$$\frac{d\hat{p}^b}{dn} = \frac{1}{Det}\left(\frac{\gamma\exp\left\{-\hat{p}^b/\mu\right\}^2}{\left[1+(n-1)\exp\left\{-\hat{p}^b/\mu\right\}\right]^3}\right)\left(\frac{\left[1+(n-1)\exp\left\{-\hat{p}^b/\mu\right\}\right]^2}{\exp\left\{-\hat{p}^b/\mu\right\}} - \epsilon_s\right).$$

Using the inequality of arithmetic and geometric means, we can bound $\frac{\left[1+(n-1)\exp\left\{-\hat{p}^b/\mu\right\}\right]^2}{\exp\left\{-\hat{p}^b/\mu\right\}} \geq 4(n-1)$, so that condition (33) implies the result. ■

## A.7 Results for logit-exponential example

We first prove the surplus implications of platform entry stated in Section 4.2. Given the closed-form solution (10) and the logit buyer quasi-demand form, we can express equilibrium buyer and seller surpluses as

$$\begin{aligned} BS &= \mu\ln\left(\exp\{\beta_0/\mu\} + n\exp\left\{-\hat{p}^b/\mu\right\}\right)(1-G(\hat{p}^s)) \\ &= \mu\ln\left(\frac{\exp\{\beta_0/\mu\}}{\theta\mu}\right)(1-G(\hat{p}^s)) \end{aligned}$$

and

$$SS = \left(\int_{\hat{p}^s}^{\infty}(v-\hat{p}^s)\,dG(v)\right)(1-\theta\mu)$$

The total surplus is

$$\begin{aligned} TS &= BS + SS + n\Pi_i^* \\ &= BS + \left(\int_{\hat{p}^s}^{\infty}\left(v+\hat{p}^b-c\right)dG(v)\right)(1-\theta\mu). \end{aligned}$$

From (10), an increase in $n$ decreases $\hat{p}^s$ and increases $\hat{p}^b$, thus raising $BS$, $SS$, and $TS$.

We now prove Corollary 2. In the partial multihoming model in Section 5, if the parameters satisfy $(n-1)(1-\lambda) < n\theta\mu < n$, then the equilibrium fees are

$$\begin{aligned} \hat{p}^b &= \mu\ln\left(\frac{n\theta\mu-(n-1)(1-\lambda)}{(1-\theta\mu)\exp\{\beta_0/\mu\}}\right) \\ \hat{p}^s &= c + \frac{(1-\lambda+\lambda n)\mu}{\theta\mu-\lambda+\lambda n} - \hat{p}^b. \end{aligned}$$

Taking derivative with respect to $n$:

$$\frac{d\hat{p}^b}{dn} = \frac{\mu(\lambda-1+\theta\mu)}{n\theta\mu-(n-1)(1-\lambda)} > 0$$

if and only if $\lambda > \bar{\lambda}_1 \equiv 1 - \theta\mu$; while

$$\frac{d\hat{p}^s}{dn} = -\frac{\lambda\mu\left(1-\theta\mu\right)}{\left(\lambda n - \lambda + \theta\mu\right)^2} - \frac{\mu\left(\lambda - 1 + \theta\mu\right)}{n\theta\mu - \left(n-1\right)\left(1-\lambda\right)}.$$

Clearly, $\frac{d\hat{p}^s}{dn}|_{\lambda=0} > 0$ and $\frac{d\hat{p}^s}{dn}|_{\lambda>\bar{\lambda}_1} < 0$. It remains to show $\frac{d\hat{p}^s}{dn}$ is single-crossing in $\lambda$, which can be easily verified by checking that

$$\frac{\mu}{\left(\lambda n - \lambda + \theta\mu\right)^2}\frac{d\hat{p}^s}{dn} = -\lambda\left(1-\theta\mu\right) - \frac{\left(\lambda - 1 + \theta\mu\right)\left(\lambda n - \lambda + \theta\mu\right)^2}{n\theta\mu - \left(n-1\right)\left(1-\lambda\right)}$$

is decreasing in $\lambda$. Hence, intermediate value theorem implies the existence of the required unique threshold $\bar{\lambda}_2 \in \left(0, \bar{\lambda}_1\right)$ where $\frac{d\hat{p}^s}{dn} < 0$ if and only if $\lambda > \bar{\lambda}_2$.

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# Multihoming and oligopolistic platform competition: online appendix

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This online appendix contains proofs of omitted results and details from the main paper.

## B Further properties of the baseline demand function

We examine the continuity and differentiability of the demand function $Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right)$ in (20).

**Claim B.1** *For any $\hat{\mathbf{p}}$, $Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right)$ is continuous in $p_i^b$ and $p_i^s$.*

**Proof.** Continuity with respect to $p_i^b$ is obvious. To show continuity with respect to $p_i^s$, note from (19) that $\lim_{p_i^s \to \hat{p}^{s-}} \hat{v}_m = \hat{p}^s$ for $m = 2, ..., n$. Similarly, note from Lemma 1 that $\lim_{p_i^s \to \hat{p}^{s+}} \hat{v} = \hat{p}^s$. Thus,

$$\begin{aligned}
& \lim_{p_i^s \to \hat{p}^{s-}} Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) \\
= & \left[1 - G\left(\hat{p}^s\right)\right] B_i^{(\mathbf{N})} + \sum_{m=0}^{n-2} \left[G\left(\hat{p}^s\right) - G\left(\hat{p}^s\right)\right] B_i^{(\mathbf{N}_{i,m})} \\
= & \left[1 - G\left(\hat{p}^s\right)\right] B_i^{(\mathbf{N})} \\
= & \lim_{p_i^s \to \hat{p}^{s+}} Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right),
\end{aligned}$$

so $Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right)$ is continuous for all $p_i^b$ and $p_i^s$, which includes $\left(p_i^b, p_i^s\right) = \hat{p}$. ■

**Claim B.2** *For any $\hat{\mathbf{p}}$,*

$$\lim_{p_i^s \to \hat{p}^{s-}} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right) \geq \lim_{p_i^s \to \hat{p}^{s+}} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right).$$

*Equality holds if in addition (i) $n = 2$, or (ii) $F$, $F_0 \sim$Gumbel$(\mu)$.*

**Proof.** Consider first $p_i^s \geq \hat{p}^s$. Then the right-hand side derivative is

$$\begin{aligned}
\lim_{p_i^s \to \hat{p}^{s+}} \frac{dQ_i}{dp_i^s}\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) &= \lim_{p_i^s \to \hat{p}^{s+}} -\frac{d\hat{v}}{dp_i^s} B_i^{(\mathbf{N})} g\left(p_i^s\right) \\
&= \lim_{p_i^s \to \hat{p}^{s+}} \frac{-B_i^{(\mathbf{N})}}{\sum_{j \in \mathbf{N}, j \neq i}\left(B_j^{(\mathbf{N})} - B_j^{(\mathbf{N}_{-i})}\right) + B_i^{(\mathbf{N})}} B_i^{(\mathbf{N})} g\left(p_i^s\right) \\
&= \frac{-B_i^{(\mathbf{N})}}{\sum_{j \in \mathbf{N}, j \neq i}\left(B_j^{(\mathbf{N})} - B_j^{(\mathbf{N}_{-i})}\right) + B_i^{(\mathbf{N})}} B_i^{(\mathbf{N})} g\left(\hat{p}^s\right).
\end{aligned}$$

Evaluating the above at $p_i^b = \hat{p}^b$, all platforms become symmetry so that functions $B_j^{(\Theta)}$ are the same for any set $\Theta$ and any given $j \in \Theta$. So, for simplicity we denote any such generic term as $B^{(\Theta)}$. Hence we have

$$\lim_{p_i^s \to \hat{p}^{s+}} \frac{dQ_i}{dp_i^s}\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) = \frac{-B^{(\mathbf{N})} B^{(\mathbf{N})}}{nB^{(\mathbf{N})} - (n-1) B^{(\mathbf{N}_{-i})}} g\left(\hat{p}^s\right). \tag{B.1}$$

When $p_i^s > \hat{p}^s$, the left hand side derivative is

$$\begin{aligned} & \lim_{p_i^s \to \hat{p}^{s-}} \frac{dQ_i}{dp_i^s}\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) \\ = & \lim_{p_i^s \to \hat{p}^{s-}} \sum_{m=0}^{n-1}\left[\frac{d\hat{v}_{m+1}}{dp_i^s} g\left(\bar{v}_{m+1}\right) - \frac{d\hat{v}_m}{dp_i^s} g\left(\bar{v}_m\right)\right] B_i^{(\mathbf{N}_{i,m})} \\ = & \; g\left(\hat{p}^s\right)\left[\sum_{m=0}^{n-1}\left[\frac{d\hat{v}_{m+1}}{dp_i^s} - \frac{d\hat{v}_m}{dp_i^s}\right] B^{(\mathbf{N}_{i,m})}\right], \end{aligned} \tag{B.2}$$

where $\frac{d\hat{v}_n}{dp_i^s} = 0$ because $\hat{v}_n \equiv \bar{v}$, $\frac{d\hat{v}_0}{dp_i^s} = 1$ since $\hat{v}_0 \equiv p_i^s$, while

$$\begin{aligned} \frac{d\hat{v}_m}{dp_i^s} &= \frac{B_i^{(\mathbf{N}_{i,m})} - B_i^{(\mathbf{N}_{i,m-1})}}{B_i^{(\mathbf{N}_{i,m})} - B_i^{(\mathbf{N}_{i,m-1})} + mB_j^{(\mathbf{N}_{i,m})} - (m-1) B_j^{(\mathbf{N}_{i,m-1})}} \text{ for } m = 1, ..., n-1 \\ \frac{d\hat{v}_m}{dp_i^s}|_{p_i^b = \hat{p}^b} &= \frac{B^{(\mathbf{N}_{i,m})} - B^{(\mathbf{N}_{i,m-1})}}{(m+1) B^{(\mathbf{N}_{i,m})} - mB^{(\mathbf{N}_{i,m-1})}}, \end{aligned}$$

in which $B^{(\cdot)}$ is as denoted earlier due to symmetry. Hence, evaluating at $p_i^b = \hat{p}^b$, ($B.2$) can be expanded

$$\begin{aligned} & \frac{1}{g\left(\hat{p}^s\right)} \lim_{p_i^s \to \hat{p}^{s-}} \frac{dQ_i}{dp_i^s}\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) \\ = & -\frac{d\hat{v}_{n-1}}{dp_i^s} B^{(\mathbf{N})} + \sum_{m=1}^{n-2}\left[\frac{d\hat{v}_{m+1}}{dp_i^s} - \frac{d\hat{v}_m}{dp_i^s}\right] B^{(\mathbf{N}_{i,m})} + \left(\frac{d\hat{v}_1}{dp_i^s} - 1\right) B^{(\mathbf{N}_{i,0})} \\ = & \frac{-B^{(\mathbf{N}_{i,n-1})} B^{(\mathbf{N}_{i,n-1})}}{nB^{(\mathbf{N}_{i,n-1})} - (n-1) B^{(\mathbf{N}_{i,n-2})}} \\ & + \frac{B^{(\mathbf{N}_{i,n-2})} B^{(\mathbf{N}_{i,n-1})}}{nB^{(\mathbf{N}_{i,n-1})} - (n-1) B^{(\mathbf{N}_{i,n-2})}} \\ & + \sum_{m=1}^{n-2}\left[\frac{B^{(\mathbf{N}_{i,m+1})} - B^{(\mathbf{N}_{i,m})}}{(m+2) B^{(\mathbf{N}_{i,m+1})} - (m+1) B^{(\mathbf{N}_{i,m})}} - \frac{B^{(\mathbf{N}_{i,m})} - B^{(\mathbf{N}_{i,m-1})}}{(m+1) B^{(\mathbf{N}_{i,m})} - mB^{(\mathbf{N}_{i,m-1})}}\right] B^{(\mathbf{N}_{i,m})} \\ & + \left(\frac{B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}}{2B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}} - 1\right) B^{(\mathbf{N}_{i,0})}. \end{aligned} \tag{B.3}$$

By definition, proving differentiability at $\left(p_i^b, p_i^s\right) = \hat{p}$ requires us to show

$$\lim_{p_i^s \to \hat{p}^{s-}} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right) = \lim_{p_i^s \to \hat{p}^{s+}} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right). \tag{B.4}$$

To prove this, we note that $\mathbf{N}_{i,n-1} = \mathbf{N}$ and that when all platforms are symmetry we have $\mathbf{N}_{i,n-2} = \mathbf{N}_{-i}$ (because both sets denote a set of $n-1$ symmetry platforms). Then, substituting for (B.1) we can rewrite (B.3) as

$$\lim_{p_i^s \to \hat{p}^{s-}} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right) = \lim_{p_i^s \to \hat{p}^{s+}} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right) + g\left(\hat{p}^s\right) \Phi\left(n\right),$$

where $\Phi\left(n\right)$ is defined as the last three lines of (B.3), i.e.

$$\begin{aligned} \Phi\left(n\right) \equiv & \frac{B^{(\mathbf{N}_{i,n-2})} B^{(\mathbf{N}_{i,n-1})}}{nB^{(\mathbf{N}_{i,n-1})} - (n-1) B^{(\mathbf{N}_{i,n-2})}} \\ & + \sum_{m=1}^{n-2}\left[\frac{B^{(\mathbf{N}_{i,m+1})} - B^{(\mathbf{N}_{i,m})}}{(m+2) B^{(\mathbf{N}_{i,m+1})} - (m+1) B^{(\mathbf{N}_{i,m})}} - \frac{B^{(\mathbf{N}_{i,m})} - B^{(\mathbf{N}_{i,m-1})}}{(m+1) B^{(\mathbf{N}_{i,m})} - mB^{(\mathbf{N}_{i,m-1})}}\right] B^{(\mathbf{N}_{i,m})} \\ & + \left(\frac{B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}}{2B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}} - 1\right) B^{(\mathbf{N}_{i,0})}. \end{aligned}$$

To conclude (B.4), it suffices to prove by induction that $\Phi\left(n\right) \geq 0$ for all $n \geq 2$. First, when $n = 2$ we have

$\mathbf{N}_{i,1} = \mathbf{N}$ so

$$\begin{aligned}\Phi(2) &= \frac{B^{(\mathbf{N}_{i,0})}B^{(\mathbf{N}_{i,1})}}{2B^{(\mathbf{N}_{i,1})}-B^{(\mathbf{N}_{i,0})}} + \left(\frac{B^{(\mathbf{N}_{i,1})}-B^{(\mathbf{N}_{i,0})}}{2B^{(\mathbf{N}_{i,1})}-B^{(\mathbf{N}_{i,0})}} - 1\right)B^{(\mathbf{N}_{i,0})} \\ &= \frac{2B^{(\mathbf{N}_{i,0})}B^{(\mathbf{N}_{i,1})}}{2B^{(\mathbf{N}_{i,1})}-B^{(\mathbf{N}_{i,0})}} - \left[\frac{B^{(\mathbf{N}_{i,0})}B^{(\mathbf{N}_{i,0})}}{2B^{(\mathbf{N}_{i,1})}-B^{(\mathbf{N}_{i,0})}} + B^{(\mathbf{N}_{i,0})}\right] = 0.\end{aligned}$$

Note that this also proves the first part of the claim, that is, the case of $n = 2$. By the inductive hypothesis, suppose $\Phi(n-1) \geq 0$. For $n \geq 3$, if we expand one more term from the summation in $\Phi(n)$ and rearrange terms we get

$$\begin{aligned}\Phi(n) &= \frac{B^{(\mathbf{N}_{i,n-2})}B^{(\mathbf{N}_{i,n-1})}}{nB^{(\mathbf{N}_{i,n-1})}-(n-1)B^{(\mathbf{N}_{i,n-2})}} \\ &\quad + \left[\frac{B^{(\mathbf{N}_{i,n-1})}-B^{(\mathbf{N}_{i,n-2})}}{nB^{(\mathbf{N}_{i,n-1})}-(n-1)B^{(\mathbf{N}_{i,n-2})}} - \frac{B^{(\mathbf{N}_{i,n-2})}-B^{(\mathbf{N}_{i,n-3})}}{(n-1)B^{(\mathbf{N}_{i,n-2})}-(n-2)B^{(\mathbf{N}_{i,n-3})}}\right]B^{(\mathbf{N}_{i,n-2})} \\ &\quad + \sum_{m=1}^{n-3}\left[\frac{B^{(\mathbf{N}_{i,m+1})}-B^{(\mathbf{N}_{i,m})}}{(m+2)B^{(\mathbf{N}_{i,m+1})}-(m+1)B^{(\mathbf{N}_{i,m})}} - \frac{B^{(\mathbf{N}_{i,m})}-B^{(\mathbf{N}_{i,m-1})}}{(m+1)B^{(\mathbf{N}_{i,m})}-mB^{(\mathbf{N}_{i,m-1})}}\right]B^{(\mathbf{N}_{i,m})} \\ &\quad + \frac{B^{(\mathbf{N}_{i,n-2})}B^{(\mathbf{N})}}{nB^{(\mathbf{N})}-(n-1)B^{(\mathbf{N}_{i,n-2})}} \\ &= \frac{\left(2B^{(\mathbf{N}_{i,n-1})}-B^{(\mathbf{N}_{i,n-2})}\right)B^{(\mathbf{N}_{i,n-2})}}{nB^{(\mathbf{N}_{i,n-1})}-(n-1)B^{(\mathbf{N}_{i,n-2})}} - \frac{B^{(\mathbf{N}_{i,n-2})}B^{(\mathbf{N}_{i,n-2})}}{(n-1)B^{(\mathbf{N}_{i,n-2})}-(n-2)B^{(\mathbf{N}_{i,n-3})}} + \Phi(n-1)\,. \qquad \text{(B.5)}\end{aligned}$$

By inductive hypothesis $\Phi(n-1) \geq 0$. Therefore, to prove $\Phi(n) \geq 0$, it remains to show

$$\frac{2B^{(\mathbf{N}_{i,n-1})}-B^{(\mathbf{N}_{i,n-2})}}{nB^{(\mathbf{N}_{i,n-1})}-(n-1)B^{(\mathbf{N}_{i,n-2})}} \geq \frac{B^{(\mathbf{N}_{i,n-2})}}{(n-1)B^{(\mathbf{N}_{i,n-2})}-(n-2)B^{(\mathbf{N}_{i,n-3})}}.$$

Rearranging the terms and cancelling out common coefficients, the inequality above is equivalent to

$$\begin{aligned}0 &\leq \frac{\left(B^{(\mathbf{N}_{i,n-2})}-B^{(\mathbf{N}_{i,n-1})}\right)}{B^{(\mathbf{N}_{i,n-1})}} - \frac{\left(B^{(\mathbf{N}_{i,n-3})}-B^{(\mathbf{N}_{i,n-2})}\right)}{B^{(\mathbf{N}_{i,n-3})}} \qquad \text{(B.6)} \\ &\simeq \frac{\partial B^{(\mathbf{N}_{i,k})}}{\partial k}|_{k=n-1} - \frac{\partial B^{(\mathbf{N}_{i,k})}}{\partial k}|_{k=n-3}.\end{aligned}$$

We know $\frac{\partial B}{\partial k} \leq 0$, so we simply need to show that $B$ is decreasing in $k$ with a decreasing magnitude, i.e. $B$ is convex in $k$. Recall that for $k \in \{0, ..., n-1\}$, we have

$$B^{(\mathbf{N}_{i,k})} = \int_{\underline{\epsilon}}^{\bar{\epsilon}-\hat{p}^b}\left[\frac{1-F\left(\epsilon_0+\hat{p}^b\right)^{k+1}}{k+1}\right]dF_0(\epsilon_0)\,.$$

Convexity of $B$ in $k$ then follows from the observation that $\frac{1-F\left(\epsilon_0+\hat{p}^b\right)^{k+1}}{k+1}$ is convex in $k$, and that convexity is preserved by integration when the integrand is always positive over the entire region of integration. So, $\Phi(n) \geq 0$ for all $n \geq 2$ as required. Finally, in the special case of Gumbel distribution, IIA property of logit-demand form implies that the right-hand side of (B.6) equals zero, so that $\Phi(n) = 0$ for all $n \geq 2$. ■

In order to determine the global quasi-concavity of the profit function in Section 3 for other distribution functions, we rely on numerical calculations. Details and codes of the numerical calculations are available from the authors upon request. Specifically, we focus on $n \in \{2,3,4\}$ and $c = 0.1$ and consider $F, F_0 \sim Gumbel(\mu)$ and $G \sim Normal(\mu_{norm}, \sigma)$ where the parameters are repeatedly picked in random from intervals: $\mu \in [1,4]$, and $\mu_{norm} \in [-10,10]$, $\sigma \in [1,6]$. We also repeat the same exercise with (i) $F, F_0 \sim Gumbel(\mu)$ and $G \sim Exponential(\theta)$, $\theta \in [1/2,2]$; (ii) $F, F_0 \sim Exponential(\theta_F)$ and $G \sim Exponential(\theta_G)$, $\theta_F, \theta_G \in [1/2,2]$; and $F, F_0 \sim Normal(\mu_{norm}, \sigma)$ and $G \sim Exponential(\theta)$, $\mu_{norm} \in [1,2]$, $\sigma \in [1,2]$ and $\theta \in [1/2,2]$.

In all the cases considered, the quasi-concavity assumption was satisfied, suggesting it does not require very special conditions to hold. Figure 7 below provides an example of the plot of platform profit function that shows quasi-concavity (assuming all other platforms are setting the equilibrium fees).

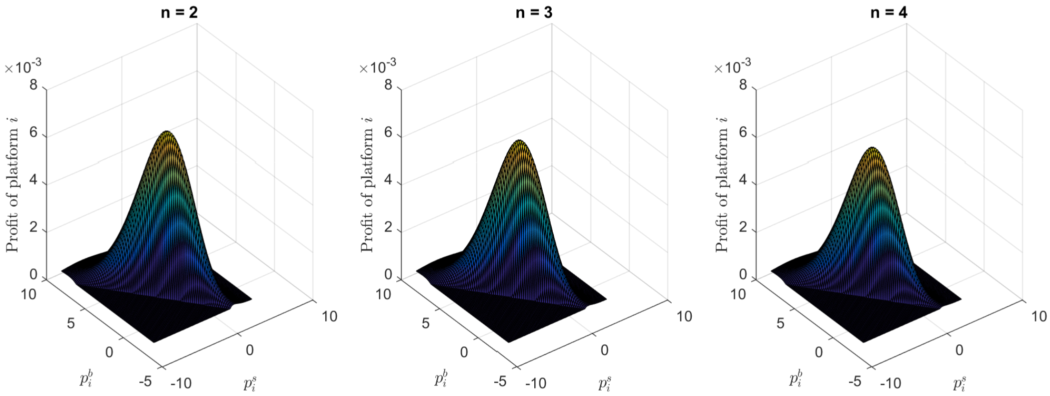


Figure 7: Profit function of firm $i$, assuming $c = 0.1$, $F$ and $F_0 \sim Gumbel(1)$, and $G \sim Normal(-4, 2)$.

# C Membership fee component

As stated in the main text, when buyers and sellers coordinate to not participating on any platform that charges strictly positive buyer or seller membership fees, then no platform has incentive to deviate from the equilibrium in Proposition 1 by charging positive membership fee component.

Meanwhile, given that all buyers are already multihoming in the equilibrium, no platform can profit from offering negative buyer membership fees. It remains to rule out deviations from the symmetric equilibrium with a negative seller fee (denoted as $P_i^s < 0$). We will focus on the case of logit buyer demand and show that, whenever a deviating platform $i$ wants to set $P_i^s < 0$ to attract additional seller participation, it is instead better off setting $p_i^s < \hat{p}^s$ to attract the same amount of participation.

Starting from the symmetric equilibrium, when $P_i^s < 0$, all sellers with $v \geq \hat{p}^s$ will continue to join all platforms while sellers with $v \in \left[\frac{P_i^s}{B_i^{(\{i\})}} + \hat{p}^s, \hat{p}^s\right]$ will singlehome on platform $i$. We reframe the platform's problem as choosing the marginal participating seller type

$$t_i^s \equiv \frac{P_i^s}{B_i^{(\{i\})}} + \hat{p}^s \leq \hat{p}^s.$$

Then, platform $i$'s profit when it charges a negative seller membership fee is

$$\tilde{\Pi}_i\left(t_i^s\right) = \left(\hat{p}^s + \hat{p}^b - c\right)\left(\left(1 - G\left(\hat{p}^s\right)\right) B_i^{(\mathbf{N})} + \left(G\left(\hat{p}^s\right) - G\left(t_i^s\right)\right) B_i^{(\{i\})}\right) + \left(t_i^s - \hat{p}^s\right)\left(1 - G\left(t_i^s\right)\right) B_i^{(\{i\})}$$

where

$$\frac{d\tilde{\Pi}_i}{dt_i^s} = \left(1 - G\left(t_i^s\right)\right) B_i^{(\{i\})} - \left(t_i^s + p_i^b - c\right) g\left(t_i^s\right) B_i^{(\{i\})}.$$

Meanwhile, from Section A.6 of the appendix, we know that when platform $i$ deviates by lowering its seller transaction fee $p_i^s < \hat{p}^s$, the marginal participating seller type is $t_i^s = p_i^s$ (and is singlehoming on platform $i$). Reframe the platform's problem as choosing the marginal participating seller type:

$$\Pi_i\left(t_i^s\right) = \left(t_i^s + \hat{p}^b - c\right)\left(\left(1 - G\left(\left(\hat{p}^s - t_i^s\right)\exp\left\{-p^b/\mu\right\} + \hat{p}^s\right)\right) B_i^{(\mathbf{N})} + \left(G\left(\left(\hat{p}^s - t_i^s\right)\exp\left\{-p_i^b/\mu\right\} + \hat{p}^s\right) - G(t_i^s)\right) B_i^{(\{i\})}\right).$$

Using $B_i^{(\mathbf{N})} < B_i^{(\{i\})}$, it is easily shown

$$\frac{d\Pi_i}{dt_i^s} < (1 - G(t_i^s))B_i^{(\{i\})} - \left(t_i^s + p_i^b - c\right) g\left(t_i^s\right) B_i^{(\{i\})}. \tag{C.1}$$

Consider a scenario where platform $i$ wants to lower the participation threshold from $t_i^s \leq \hat{p}^s$ to $t_i^s + \Delta$, where $\Delta < 0$. Whenever the platform finds it profitable to do so through a deviation with a negative seller membership fee ($\frac{d\tilde{\Pi}_i}{dt_i^s}\Delta > 0$), inequality (C.1) immediately implies

$$\frac{d\Pi_i}{dt_i^s}\Delta t_i^s > \frac{d\tilde{\Pi}_i}{dt_i^s}\Delta t_i^s > 0,$$

that is, a deviation by lowering seller transaction fee is more profitable.

# D Multihoming behaviors of buyers

This section corresponds to the equilibrium analysis of Section 5 in the main text.

**Demand derivation.** We first derive the demand functions facing each platform. Consider a deviating platform $i$ that charges $\left(p_i^b, p_i^s\right) \neq \left(\hat{p}^b, \hat{p}^s\right)$. Note that the decisions of buyers are shown in the main text and so are omitted here. Consider $p_i^s \geq \hat{p}^s$. For a seller with type $v$, we write her total surplus from joining all platforms $j \neq i$ as

$$\begin{aligned} & (v - \hat{p}^s) \sum_{j \in \mathbf{N}_{-i}} \left((1-\lambda) B_j^{(\mathbf{N})} + \lambda B_j^{(\mathbf{N}_{-i})}\right) \\ = \; & (v - \hat{p}^s) \sum_{j \in \mathbf{N}_{-i}} \tilde{B}_j^{(\mathbf{N}_{-i})}, \end{aligned} \tag{D.1}$$

where

$$\tilde{B}_j^{(\Theta)} \equiv (1-\lambda) B_j^{(\mathbf{N})} + \lambda B_j^{(\Theta)} \tag{D.2}$$

can be thought of as a "composite" buyer quasi-demand. Likewise, if the seller joins all platforms including $i$, then her total surplus is

$$\begin{aligned} & (v - \hat{p}^s) \left[(1-\lambda) \sum_{j \in \mathbf{N}_{-i}} B_j^{(\mathbf{N})} + \lambda \sum_{j \in \mathbf{N}_{-i}} B_j^{(\mathbf{N})}\right] + (v - p_i^s) \left[(1-\lambda) B_i^{(\mathbf{N})} + \lambda B_i^{(\mathbf{N})}\right] \\ = \; & (v - \hat{p}^s) \sum_{j \in \mathbf{N}_{-i}} \tilde{B}_j^{(\mathbf{N})} + (v - p_i^s) \tilde{B}_i^{(\mathbf{N})}. \end{aligned} \tag{D.3}$$

Denote

$$\tilde{\sigma}_i \equiv 1 - \frac{\sum_{j \in \mathbf{N}_{-i}} (\tilde{B}_j^{(\mathbf{N})} - \tilde{B}_j^{(\mathbf{N}_{-i})})}{\tilde{B}_i^{(\mathbf{N})}}.$$

Comparing (D.1) and (D.3), we can pin down the threshold $\tilde{v}$ as in Lemma 1:

$$\tilde{v} = \frac{p_i^s}{\tilde{\sigma}_i} - \frac{1 - \tilde{\sigma}_i}{\tilde{\sigma}_i} \hat{p}^s.$$

Likewise, when $p_i^s < \hat{p}^s$, with similar calculations we can pin down thresholds $\tilde{v}_m$ for $m = 1, ..., n-1$ as in Lemma A.1:

$$\tilde{v}_m \equiv \frac{p_i^s \left[\tilde{B}_i^{(\mathbf{N}_{i,m})} - \tilde{B}_i^{(\mathbf{N}_{i,m-1})}\right] + \hat{p}^s \left[m\tilde{B}_j^{(\mathbf{N}_{i,m})} - (m-1)\tilde{B}_j^{(\mathbf{N}_{i,m-1})}\right]}{\tilde{B}_i^{(\mathbf{N}_{i,m})} - \tilde{B}_i^{(\mathbf{N}_{i,m-1})} + m\tilde{B}_j^{(\mathbf{N}_{i,m})} - (m-1)\tilde{B}_j^{(\mathbf{N}_{i,m-1})}}.$$

We can further define $\tilde{v}_n \equiv \bar{v}$ and $\tilde{v}_0 \equiv p_i^s$. Then the formal characterization of seller participation decisions is the same as Lemma A.1 after replacing the relevant thresholds with $\hat{v}_m$.

**Equilibrium.** To derive the equilibrium fees, it suffices to focus on the seller participation profile after an upward deviation by platform $i$, that is, $p_i^s \geq \hat{p}^s$. Note

$$\begin{aligned}\Pi_i &= \left(p_i^b + p_i^s - c\right) Q_i \\ &= \left(p_i^b + p_i^s - c\right) \left(1 - G\left(\tilde{v}\right)\right) B_i^{(\mathbf{N})},\end{aligned}$$

where we use $\tilde{B}_i^{(\mathbf{N})} = B_i^{(\mathbf{N})}$ given (D.2). We assume that $\Pi_i$ is quasi-concave in $\left(p_i^b, p_i^s\right)$ so that the equilibrium can be characterized by the usual first-order condition. We numerically verified that $\Pi_i$ is quasi-concave for $\lambda \in \{0.1, 0.5, 0.9\}$ over all the distributional and parameter configurations considered in the baseline model. The details and codes of the simulations are available from the authors upon request.

The demand derivatives, after imposing symmetry, can be calculated as follows:

$$\frac{dQ_i\left(\hat{\mathbf{p}};\hat{\mathbf{p}}\right)}{dp_i^b} = \left(1 - G\left(\hat{p}^s\right)\right) \frac{\partial B_i^{(\mathbf{N})}}{\partial p_i^b}|_{\mathbf{p}=\hat{\mathbf{p}}} = -\frac{1}{X} Q_i\left(\hat{\mathbf{p}};\hat{\mathbf{p}}\right)$$

and

$$\begin{aligned}\frac{dQ_i\left(\hat{\mathbf{p}};\hat{\mathbf{p}}\right)}{dp_i^s} &= -\frac{d\tilde{v}}{dp_i^s} g\left(\hat{p}^s\right) B_i^{(\mathbf{N})}|_{\mathbf{p}=\hat{\mathbf{p}}} \\ &= -\frac{1}{\sigma_\lambda} \frac{g\left(\hat{p}^s\right)}{1 - G\left(\hat{p}^s\right)} Q_i\left(\hat{\mathbf{p}};\hat{\mathbf{p}}\right).\end{aligned}$$

Then, the standard first-order conditions yield the equilibrium in (17).

The proofs of the propositions are provided in the Appendix A.4 and A.5.

## D.1 Observable seller fee

We now extend our analysis in Section 5 to the case where buyers can observe the fees set on the seller side. We will focus on the polar cases where $\lambda \to 0$ and $\lambda \to 1$. We know that $\lambda \to 1$ corresponds to our baseline model. In what follows, we focus on the equilibrium for $\lambda \to 0$.

Denote the symmetric fee equilibrium with $\lambda \to 0$ as $\tilde{p} = \left(\tilde{p}^b, \tilde{p}^s\right)$. Consider a deviating platform $i$ that sets $p_i = \left(p_i^b, p_i^s\right) \neq \tilde{p}$, while all the remaining platforms continue to set $\tilde{p}$. We first note that the seller's participation decision is the same as in the baseline model, such that the number of sellers joining platform $i$ is $1 - G\left(p_i^s\right)$. Then, a given buyer's total utility from participating and being able to transact with sellers through platform $i$ is

$$\begin{aligned}U_i^b &= \max\left\{\epsilon_i - p_i^b, \epsilon_0\right\}\left(1 - G\left(p_i^s\right)\right) + \epsilon_0 G\left(p_i^s\right) \\ &= \max\left\{\epsilon_i - p_i^b - \epsilon_0, 0\right\}\left(1 - G\left(p_i^s\right)\right) + \epsilon_0.\end{aligned}$$

A buyer joins platform $i$ if and only if $U_i^b \geq \max_{j\in\mathbf{N}} U_j^b$, and then uses it for a transaction (with each seller) if $\epsilon_i - p_i^b > \epsilon_0$. Therefore, the total mass of buyers using platform $i$ for transactions (or buyer quasi-demand) is

$$\begin{aligned}B_i &\equiv \Pr\left(\left(\epsilon_i - p_i^b - \epsilon_0\right)\left(1 - G\left(p_i^s\right)\right) \geq \max_{j\in\mathbf{N}}\left\{\left(\epsilon_j - \tilde{p}^b - \epsilon_0\right)\left(1 - G\left(\tilde{p}^s\right)\right), 0\right\}\right) \\ &= \int_{\underline{\epsilon}}^{\bar{\epsilon}}\int_{\underline{\epsilon}}^{\bar{\epsilon}} 1 - F\left(\max\left\{\epsilon - \tilde{p}^b - \epsilon_0, 0\right\}\frac{1 - G(\tilde{p}^s)}{1 - G(p_i^s)} + p_i^b + \epsilon_0\right) dF^{n-1}(\epsilon) dF_0(\epsilon_0).\end{aligned}$$

Notice that when buyers observe the seller fee, their quasi-demand for platform $i$ is decreasing in $p_i^s$ because a higher seller fee decreases seller participation, making it less attractive to buyers.

The total demand is $Q_i(\mathbf{p}_i;\hat{\mathbf{p}}) = (1-G(p_i^s))\,B_i$. The usual demand derivative terms can be calculated as

$$-\frac{Q_i(\hat{\mathbf{p}};\hat{\mathbf{p}})}{dQ_i(\hat{\mathbf{p}};\hat{\mathbf{p}})/dp_i^b} = X(\tilde{p}^b;n)$$

and

$$\begin{aligned}-\frac{Q_i(\hat{\mathbf{p}};\hat{\mathbf{p}})}{dQ_i(\hat{\mathbf{p}};\hat{\mathbf{p}})/dp_i^s} &= \frac{1-G(\tilde{p}^s)B_i}{g(\tilde{p}^s)\left(B_i+\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\int_{p^b+\epsilon_0}^{\bar{\epsilon}} f(\epsilon)\,dF^{n-1}(\epsilon)\,dF_0(\epsilon_0)\right)}\\ &= \frac{1-G(\tilde{p}^s)}{g(\tilde{p}^s)}\delta(\tilde{p}^b;n),\end{aligned}$$

where we have defined, for any arbitrarily given (symmetric) equilibrium buyer fee $p^b$,

$$\delta(p^b;n) \equiv \left(1+\frac{\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\int_{p^b+\epsilon_0}^{\bar{\epsilon}} f(\epsilon)\,dF^{n-1}(\epsilon)\,dF_0(\epsilon_0)}{\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left(1-F\left(\max\{\epsilon,p^b+\epsilon_0\}\right)\right)dF^{n-1}(\epsilon)dF_0(\epsilon_0)}\right)^{-1}. \tag{D.4}$$

Here, $\delta(p^b;n)$ is an inverse measure of how changes in seller participation affect the total demand for platform $i$. The first component in (D.4) reflects that, each additional seller participating increases the number of transactions that can be made by inframarginal buyers who have participated on platform $i$. The second component in (D.4) reflects that, each additional seller participating also makes platform $i$ more attractive to buyers, expanding the number of buyers that participate on platform $i$.

Provided that the profit function $\Pi_i = \left(p_i^b+p_i^s-c\right)Q_i(\mathbf{p}_i;\hat{\mathbf{p}})$ is quasiconcave in $\mathbf{p}_i$, the usual first-order condition shows that a pure symmetric pricing equilibrium is characterized by all platforms setting $\tilde{\mathbf{p}} = (\tilde{p}^b,\tilde{p}^s)$ which solves

$$\tilde{p}^b+\tilde{p}^s-c = X(\tilde{p}^b;n) = \frac{1-G(\tilde{p}^s)}{g(\tilde{p}^s)}\delta(\tilde{p}^b;n). \tag{D.5}$$

## D.2 Comparative statics with observable seller fees

To derive further results, we focus on the special case in which $F$, $F_0 \sim$ Gumbel with scale parameter $\mu$. In this case, it can be shown that

$$\delta(\tilde{p}^b;n) = \frac{1+n\exp\left\{-\tilde{p}^b/\mu\right\}}{1+\left(n+\frac{n-1}{\mu}\right)\exp\{-\tilde{p}^b/\mu\}},$$

which is decreasing in $n$ and increasing in $\tilde{p}^b$. Meanwhile, the expressions for $X\left(p^b;n\right)$ and $\sigma\left(p^b;n\right)$ follow from (9) in Section A.6.

The following proposition corresponds to Proposition 3. We compare the equilibrium with $\lambda\to 1$ ($\hat{p}^b$ and $\hat{p}^s$) and the equilibrium with $\lambda \to 0$ ($\tilde{p}^b$ and $\tilde{p}^s$).

**Proposition D.1** *(Effect of buyer multihoming) Suppose $\mu \geq 1$. A change from $\lambda\to 0$ to $\lambda \to 1$ decreases total fees ($\hat{p}^b+\hat{p}^s < \tilde{p}^b+\tilde{p}^s$), decreases the seller fee ($\hat{p}^s<\tilde{p}^s$), and increases the buyer fee ($\hat{p}^b > \tilde{p}^b$).*

**Proof.** First, it is straightforward to verify that $\mu \geq 1$ implies $n\exp\left\{-\tilde{p}^b/\mu\right\} \geq \frac{1}{\mu}-1$, which implies

$$\sigma(\tilde{p}^b;n) = \frac{1}{1+(n-1)\exp\{-\tilde{p}^b/\mu\}} < \frac{1+n\exp\left\{-\tilde{p}^b/\mu\right\}}{1+\left(n+\frac{n-1}{\mu}\right)\exp\{-\tilde{p}^b/\mu\}} = \delta(\tilde{p}^b;n).$$

From the respective equilibrium conditions, $\sigma(\tilde{p}^b;n) < \delta(\tilde{p}^b;n)$ immediately implies $\hat{p}^b + \hat{p}^s < \tilde{p}^b + \tilde{p}^s$. Next, define the function $P^s\left(p^b\right) = X(p^b;n) - p^b + c$, which is decreasing in $p^b$. Notice $\tilde{p}^s = P^s\left(\tilde{p}^b\right)$ and $\hat{p}^s = P^s\left(\hat{p}^b\right)$. Using this notation, $\tilde{p}^b$ is pinned down by

$$X(\tilde{p}^b;n)\frac{g\left(P^s\left(\tilde{p}^b\right)\right)}{1-G\left(P^s\left(\tilde{p}^b\right)\right)} - \delta(\tilde{p}^b;n) = 0.$$

We know $\sigma(\tilde{p}^b;n) < \delta(\tilde{p}^b;n)$, so

$$X(\tilde{p}^b;n)\frac{g\left(P^s\left(\tilde{p}^b\right)\right)}{1-G\left(P^s\left(\tilde{p}^b\right)\right)} - \sigma(\tilde{p}^b;n) > 0.$$

Notice the left-hand side of this expression is decreasing in $p^b$. To show $\tilde{p}^b < \hat{p}^b$, suppose by contradiction $\tilde{p}^b \geq \hat{p}^b$. Then it implies

$$X(\hat{p}^b;n)\frac{g\left(P^s\left(\hat{p}^b\right)\right)}{1-G\left(P^s\left(\hat{p}^b\right)\right)} - \sigma(\hat{p}^b;n) \geq X(\tilde{p}^b;n)\frac{g\left(P^s\left(\tilde{p}^b\right)\right)}{1-G\left(P^s\left(\tilde{p}^b\right)\right)} - \sigma(\tilde{p}^b;n) > 0,$$

contradicting the definition of $\hat{p}^b$. Therefore, we must have $\tilde{p}^b < \hat{p}^b$, which immediately implies $\hat{p}^s < \tilde{p}^s$. ■

The next proposition corresponds to the second part of Proposition 4 in the main text (recall that the first part of the proposition is exactly Proposition 2).

**Proposition D.2** *(Increased platform competition) Suppose buyers observe seller fees. In the equilibrium characterized by (D.5), an increase in $n$ (i.e. platform entry) decreases the total fee $\tilde{p}^s + \tilde{p}^b$. Furthermore, an increase in $n$ decreases the buyer fee $\tilde{p}^b$ if $\exp\left\{-\tilde{p}^b/\mu\right\} > \frac{1}{\mu}$, and increases the seller fee $\tilde{p}^s$ if in addition $\exp\left\{-\tilde{p}^b/\mu\right\} > \Gamma$, where $\Gamma$ is a threshold defined in (D.8) and $\Gamma$ is decreasing in $\mu$.*

**Proof.** Denote $M\left(\hat{p}^s\right) \equiv \frac{1-G(\hat{p}^s)}{g(\hat{p}^s)}$. Total differentiation on the equilibrium (D.5), in matrix form, gives

$$\begin{bmatrix} 1-X' & 1 \\ 1-M\frac{\partial\delta}{\partial\tilde{p}^b} & 1-\delta\frac{\partial M}{\partial\tilde{p}^s} \end{bmatrix} \begin{bmatrix} \frac{d\tilde{p}^b}{dn} \\ \frac{d\tilde{p}^s}{dn} \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial n} \\ M\frac{\partial\delta}{\partial n} \end{bmatrix}. \tag{D.6}$$

Since $X' \leq 0$, $\frac{\partial M}{\partial\tilde{p}^s} < 0$, so accordingly the matrix in (D.6) has determinant

$$Det \equiv \underbrace{(1-X')\left(1-\delta\frac{\partial M}{\partial\tilde{p}^s}\right)}_{>1} - 1 + M\underbrace{\frac{\partial\delta}{\partial\tilde{p}^b}}_{>0} > 0.$$

By Cramer's rule,

$$\frac{d\tilde{p}^s}{dn} = \frac{1}{Det}\begin{vmatrix} 1-X' & \frac{\partial X}{\partial n} \\ 1-M\frac{\partial\delta}{\partial\tilde{p}^b} & M\frac{\partial\delta}{\partial n} \end{vmatrix} = \frac{1}{Det}\left[\left(M\frac{\partial\delta}{\partial n} - \frac{\partial X}{\partial n}\right) + M\left(\underbrace{\frac{\partial\delta}{\partial\tilde{p}^b}\frac{\partial X}{\partial n}}_{<0} - \underbrace{\frac{\partial\delta}{\partial n}X'}_{\geq 0}\right)\right];$$

$$\frac{d\tilde{p}^b}{dn} = \frac{1}{Det}\begin{vmatrix} \frac{\partial X}{\partial n} & 1 \\ M\frac{\partial\sigma}{\partial n} & 1-\sigma\frac{\partial M}{\partial\tilde{p}^s} \end{vmatrix} = \frac{1}{Det}\left[\left(\frac{\partial X}{\partial n} - M\frac{\partial\delta}{\partial n}\right) - \delta\underbrace{\frac{\partial M}{\partial\tilde{p}^s}\frac{\partial X}{\partial n}}_{\geq 0}\right].$$

Clearly, $\frac{d\tilde{p}^b}{dn}+\frac{d\tilde{p}^s}{dn}<0$. Next, after appropriate substitutions, we can arrive at

$$\begin{aligned} & M\frac{\partial\delta}{\partial n}-\frac{\partial X}{\partial n} \\ = & \frac{\mu\exp\left\{-\tilde{p}^b/\mu\right\}}{1+(n-1)\exp\left\{-\tilde{p}^b/\mu\right\}}\left(\frac{\exp\left\{-\tilde{p}^b/\mu\right\}}{1+(n-1)\exp\left\{-\tilde{p}^b/\mu\right\}}-\frac{1+\exp\left\{-\tilde{p}^b/\mu\right\}}{\mu+(\mu n+n-1)\exp\left\{-\tilde{p}^b/\mu\right\}}\right), \end{aligned}$$

which is positive if and only if $\exp\left\{-\tilde{p}^b/\mu\right\}>\frac{1}{\mu}$. Therefore, $\frac{d\tilde{p}^b}{dn}<0$ if $\exp\left\{-\tilde{p}^b/\mu\right\}>\frac{1}{\mu}$ holds. Similarly, we can calculate

$$\begin{aligned} & M\left(\frac{\partial\delta}{\partial\hat{p}^b}\frac{\partial X}{\partial n}-\frac{\partial\delta}{\partial n}\frac{\partial X}{\partial\hat{p}^b}\right) \\ = & -\frac{\mu\exp\left\{-\tilde{p}^b/\mu\right\}^2\left[1+(n-1)\exp\left\{-\tilde{p}^b/\mu\right\}+\exp\left\{-\tilde{p}^b/\mu\right\}\right]}{\left[1+(n-1)\exp\left\{-\tilde{p}^b/\mu\right\}\right]^3\left[\mu+(n+\mu n-1)\exp\left\{-\tilde{p}^b/\mu\right\}\right]}. \end{aligned}$$

Substituting these expressions and rearranging, we get $\frac{d\tilde{p}^s}{dn}>0$ if and only if

$$\frac{2}{\exp\left\{-\tilde{p}^b/\mu\right\}}+\frac{1}{n+1}\left(\frac{1}{\exp\left\{-\tilde{p}^b/\mu\right\}}+1\right)^2-\mu-(n-1)\left(\mu\exp\left\{-\tilde{p}^b/\mu\right\}-1\right)<0. \tag{D.7}$$

Notice that the left-hand side of (D.7) is decreasing in $\exp\left\{-\tilde{p}^b/\mu\right\}$. Therefore, an application of the intermediate value theorem implies that there exists a unique cutoff $\Gamma$, defined implicitly by

$$\frac{2}{\Gamma}+\frac{1}{n+1}\left(\frac{1}{\Gamma}+1\right)^2-\mu-(\mu\Gamma-1)(n-1)=0, \tag{D.8}$$

such that (D.7) holds if and only if $\exp\left\{-\tilde{p}^b/\mu\right\}>\Gamma$. Notice that $\Gamma$ is decreasing in $\mu$ and $n$. ■

To the extent that $\exp\left\{-\tilde{p}^b/\mu\right\}$ is bounded below (e.g. when $\tilde{p}^b\leq 0$ so that $\exp\left\{-\tilde{p}^b/\mu\right\}\geq 1$), then the conditions in Proposition D.2 hold whenever $\mu$ is sufficiently large (i.e. platforms are sufficiently differentiated from buyers' perspective).

# E Value of transactions and user heterogeneity

This section corresponds to Section 6 in the main text. Throughout, we denote

$$M\left(\hat{p}^s\right)\equiv\left(\frac{1-G(\frac{\hat{p}^s-\alpha^s}{\gamma^s})}{g(\frac{\hat{p}^s-\alpha^s}{\gamma^s})}\right)\gamma^s,$$

where $\frac{\partial M}{\partial\hat{p}^s}<0$ by log-concavity of $1-G$.

**Proof of Proposition 5.** By linearity, note that $\frac{\partial X}{\partial\alpha^b}=-\frac{\partial X}{\partial\hat{p}^b}=-\frac{1}{\gamma^b}X'\geq 0$ and $\frac{\partial\sigma}{\partial\alpha^b}=-\frac{\partial\sigma}{\partial\hat{p}^b}=-\frac{1}{\gamma^b}\sigma'<0$, where $X'\leq 0$ and $\sigma'>0$ are the derivatives of $X$ and $\sigma$ with respect to the first argument (Lemma A.2 and Lemma A.3). Total differentiation of the equilibrium (18), in matrix form, gives

$$\begin{bmatrix} 1-X' & 1 \\ 1-\frac{M}{\gamma^b}\sigma' & 1-\sigma\frac{\partial M}{\partial\hat{p}^s} \end{bmatrix}\begin{bmatrix} \frac{d\hat{p}^b}{d\alpha^b} \\ \frac{d\hat{p}^s}{d\alpha^b} \end{bmatrix}=\begin{bmatrix} -X' \\ -\frac{M}{\gamma^b}\sigma' \end{bmatrix}. \tag{E.1}$$

The matrix in (E.1) has a strictly positive determinant

$$Det=-\left(1-X'\right)\sigma\frac{\partial M}{\partial\hat{p}^s}+\frac{M}{\gamma^b}\sigma'-X'>0,$$

By Cramer's rule,

$$\frac{d\hat{p}^s}{d\alpha^b} = \frac{-1}{Det}\left[\frac{M}{\gamma^b}\sigma' - X'\right] = -\left(1 + \frac{(1-X')}{Det}\sigma\frac{\partial M}{\partial \hat{p}^s}\right) \in (-1,0) \tag{E.2}$$

$$\frac{d\hat{p}^b}{d\alpha^b} = \frac{1}{Det}\left[\left(\frac{M}{\gamma^b}\sigma' - X'\right) + \sigma\frac{\partial M}{\partial \hat{p}^s}X'\right] = 1 + \frac{1}{Det}\sigma\frac{\partial M}{\partial \hat{p}^s} \in (0,1) \tag{E.3}$$

$$\frac{d\hat{p}^s}{d\alpha^b} + \frac{d\hat{p}^b}{d\alpha^b} = \frac{1}{Det}\sigma\frac{\partial M}{\partial \hat{p}^s}X' \geq 0.$$

The results of $\frac{d\hat{p}^b}{d\alpha^s} < 0$, $\frac{d\hat{p}^s}{d\alpha^s} > 0$, and $\frac{d\hat{p}^s}{d\alpha^b} + \frac{d\hat{p}^b}{d\alpha^b} \geq 0$ can be proven similarly by applying Cramer's rule to the following matrix:

$$\begin{bmatrix} 1-X' & 1 \\ 1-\frac{M}{\gamma^b}\sigma' & 1-\sigma\frac{\partial M}{\partial \hat{p}^s} \end{bmatrix}\begin{bmatrix} \frac{d\hat{p}^b}{d\alpha^s} \\ \frac{d\hat{p}^s}{d\alpha^s} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{\partial M}{\partial \hat{p}^s}\sigma \end{bmatrix},$$

where

$$\frac{d\hat{p}^s}{d\alpha^s} = \frac{1}{Det}\left[\frac{\partial M}{\partial \hat{p}^s}\sigma\right] < 0$$

$$\frac{d\hat{p}^b}{d\alpha^s} = \frac{-1}{Det}(1-X')\frac{\partial M}{\partial \hat{p}^s}\sigma = \frac{-(1-X')\frac{\partial M}{\partial \hat{p}^s}\sigma}{-(1-X')\sigma\frac{\partial M}{\partial \hat{p}^s} + \frac{M}{\gamma^b}\sigma' - X'} \in (0,1).$$

**Proof of Proposition 6.** To prove the first result, we note from the proof of Proposition 5 that $\hat{p}^b - \alpha^b$ is strictly decreasing in $\alpha^b$ while $\hat{p}^s + \alpha^b$ is strictly increasing in $\alpha^b$. Thus, if $\alpha^b$ is sufficiently small, we have $\hat{p}^b - \alpha^b \geq 0$ and $\hat{p}^s + \alpha^b \leq 0$. From equilibrium (18),

$$\frac{\partial X}{\partial \gamma^b} = -\left(\frac{\hat{p}^b - \alpha^b}{\gamma^{b2}}\right)X' \geq 0 \quad \text{and} \quad \frac{\partial \sigma}{\partial \gamma^b} = -\left(\frac{\hat{p}^b - \alpha^b}{\gamma^{b2}}\right)\sigma' \leq 0.$$

Total differentiation of the equilibrium (18), in matrix form, gives

$$\begin{bmatrix} 1-X' & 1 \\ 1-\frac{M}{\gamma^b}\sigma' & 1-\sigma\frac{\partial M}{\partial \hat{p}^s} \end{bmatrix}\begin{bmatrix} \frac{d\hat{p}^b}{d\gamma^b} \\ \frac{d\hat{p}^s}{d\gamma^b} \end{bmatrix} = \begin{bmatrix} X + \gamma^b\frac{\partial X}{\partial \gamma^b} \\ M\ \frac{\partial \sigma}{\partial \gamma^b} \end{bmatrix},$$

where the matrix on the left hand side is the same as in (E.1). By Cramer's rule,

$$\begin{aligned}
\frac{d\hat{p}^b}{d\gamma^b} &= \frac{1}{Det}\left[\left(X + \gamma^b\frac{\partial X}{\partial \gamma^b}\right)\left(1-\sigma\frac{\partial M}{\partial \hat{p}^s}\right) - M\ \frac{\partial \sigma}{\partial \gamma^b}\right] \geq 0 \\
\frac{d\hat{p}^s}{d\gamma^b} &= \frac{1}{Det}\left[(1-X')M\ \frac{\partial \sigma}{\partial \gamma^b} - \left(1-\frac{M}{\gamma^b}\sigma'\right)\left(X + \gamma^b\frac{\partial X}{\partial \gamma^b}\right)\right] \\
&= \frac{1}{Det}\left[-\left(X + \gamma^b\frac{\partial X}{\partial \gamma^b}\right) + M\ \frac{\partial \sigma}{\partial \gamma^b} + \frac{M}{\gamma^b}\sigma' X\right] \\
&= \frac{1}{Det}\left[-\left(X + \gamma^b\frac{\partial X}{\partial \gamma^b}\right) + \sigma'\frac{M}{(\gamma^b)^2}\left(\hat{p}^s + \alpha^b - c\right)\right] \leq 0,
\end{aligned}$$

where the last equality uses the equilibrium condition $\hat{p}^b + \hat{p}^s - c = \gamma^b X$. Finally,

$$\frac{d\hat{p}^b}{d\gamma^b} + \frac{d\hat{p}^s}{d\gamma^b} = \frac{1}{Det}\left[M\sigma'\frac{X}{\gamma^b} - \sigma\frac{\partial M}{\partial \hat{p}^s}\left(X + \gamma^b\frac{\partial X}{\partial \gamma^b}\right)\right] \geq 0.$$

To prove the second result, we note from the proof of Proposition 5 that $\hat{p}^s - \alpha^s$ is strictly decreasing in $\alpha^s$. Thus, if $\alpha^s$ is sufficiently small, we have $\hat{p}^s - \alpha^s \geq 0$ so that

$$\frac{\partial M}{\partial \gamma^s} = \frac{1 - G(\frac{\hat{p}^s - \alpha^s}{\gamma^s})}{g(\frac{\hat{p}^s - \alpha^s}{\gamma^s})} + \frac{1 - G(\frac{\hat{p}^s - \alpha^s}{\gamma^s})}{g(\frac{\hat{p}^s - \alpha^s}{\gamma^s})} \left( \frac{\hat{p}^s - \alpha^s}{\gamma^s} \right) \geq 0.$$

Applying Cramer's rule to the following matrix yields the stated results.

$$\begin{bmatrix} 1 - X' & 1 \\ 1 - \frac{M}{\gamma^b}\sigma' & 1 - \sigma \frac{\partial M}{\partial \hat{p}^s} \end{bmatrix} \begin{bmatrix} \frac{d\hat{p}^b}{d\gamma^s} \\ \frac{d\hat{p}^s}{d\gamma^s} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\partial M}{\partial \gamma^s}\sigma \end{bmatrix}.$$